Physics Formulary

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The Greek alphabet:

A	α	Alpha	H	η	Eta	N	ν	Nu	T	τ	Tau
B	β	Beta	Θ	θ, ϑ	Theta	Ξ	ξ	Xi	Y	υ	Upsilon
Γ	γ	Gamma	I	ι	Iota	O	o	Omicron	Φ	ϕ, φ	Phi
Δ	δ	Delta	K	κ	Kappa	Π	π	Pi	X	χ	Chi
E	ϵ	Epsilon	Λ	λ	Lambda	P	ρ, ϱ	Rho	Ψ	ψ	Psi
Z	ζ	Zeta	M	μ	Mu	Σ	σ, ς	Sigma	Ω	ω	Omega

1 Units, orders of magnitude and constants

Units

symbol	**name**	**unity**	
t, T	**t**ime	s	second
r, $\vec{r}$, x	position	m	meter
Δr, $\Delta\vec{r}$, Δx, s	displacement	m	meter
v, $\vec{v}$, c	speed, **v**elocity, speed of light	$\frac{\mathrm{m}}{\mathrm{s}}$	meter per second
p, $\vec{p}$	momentum	$\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}}$	kilogram · meter per second
a, $\vec{a}$, g	**a**cceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$	meter per square second
m	**m**ass	kg	kilogram
F, $\vec{F}$	**F**orce	N	Newton
A	**A**rea	m^2	square meter
V	**V**olume	m^3	cubic meter
p	**p**ressure	Pa	Pascal
ρ	density	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	kilogram per cubic meter
W, E, U, K, Q	**W**ork, **E**nergy, heat	J	Joule
P	**P**ower	W	Watt
T, ϑ	**T**emperature	K, °C	Kelvin, Celsius
f	**f**requency	Hz	Hertz = second^{-1}
ω, $\vec{\omega}$	angular velocity	$\frac{1}{\mathrm{s}}$	rad per second
τ, $\vec{\tau}$	torque	Nm	Newton · meter
L, $\vec{L}$	angular momentum	$\mathrm{kg}\,\frac{\mathrm{m}^2}{\mathrm{s}}$	kilogram · square meter per second
I, Θ	**I**nertia	$\mathrm{kg}\,\mathrm{m}^2$	kilogram · square meter
Q, q	charge	C	Coulomb
V	voltage	V	Volt
I	current	A	Ampere
R	**R**esistance	Ω	Ohm
C	**C**apacitance	F	Farad
L	inductance	H	Henry
B	magnetic field	T	Tesla

Orders of magnitude, multiplicators

name	symbol	multiplicator
Tera	T	$\cdot 10^{12}$
Giga	G	$\cdot 10^{9}$
Mega	M	$\cdot 10^{6}$
Kilo	k	$\cdot 10^{3} = \cdot 1000$
Hecto	h	$\cdot 10^{2} = \cdot 100$

name	symbol	multiplicator
deci	d	$\cdot 10^{-1} = \cdot 0.1$
centi	c	$\cdot 10^{-2} = \cdot 0.01$
milli	m	$\cdot 10^{-3} = \cdot 0.001$
micro	μ	$\cdot 10^{-6}$
nano	n	$\cdot 10^{-9}$
pico	p	$\cdot 10^{-12}$

Physical constants

Gravitational constant	$G = 6.67 \cdot 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Speed of light in vaccum	$c = 2.998 \cdot 10^{8}\ \mathrm{m\,s^{-1}}$
Acceleration of gravity on Earth	$g = 9.8\ \mathrm{m\,s^{-2}}$
Electron rest mass	$m_e = 9.109 \cdot 10^{-31}\ \mathrm{kg}$
Proton rest mass	$m_p = 1.672$[illegible] $\cdot 10^{-27}\ \mathrm{kg}$
Neutron rest mass	$m_n = 1.$[illegible]$749 \cdot 10^{-27}\ \mathrm{kg}$
Atomic mass unit	$u = 1.660$[illegible] $\cdot 10^{-27}\ \mathrm{kg}$
Elementary charge	$e = 1.602 \cdot 10^{-19}\ \mathrm{C}$
Permeability of the vacuum	$\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{V\,s\,A^{-1}\,m^{-1}}$
Permittivity of the vacuum	$\epsilon_0 = \frac{1}{\mu_0 c^2} = 8.854 \cdot 10^{-12}\ \mathrm{A\,s\,V^{-1}\,m^{-1}}$
Solar constant	$S = 1360\ \mathrm{Wm^{-2}}$
Hubble's constant	$H_0 = 70.8 \pm 4.0\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$
normal pressure	$p_0 = 1.013 \cdot 10^{5}\ \mathrm{N\,m^{-2}} = 1.013\ \mathrm{bar}$
normal temperature	$T_0 = 273.15\ \mathrm{K} = 0\,^\circ\mathrm{C}$
normal volume of an ideal gas	$V_0 = 22.414 \cdot 10^{-3}\ \mathrm{m^3\,mol^{-1}}$
Boltzmann's constant	$k_B = 1.38 \cdot 10^{-23}\ \mathrm{J\,K^{-1}}$
Avogadro's constant	$N_A = 6.022 \cdot 10^{23}\ \mathrm{mol^{-1}}$
Molar gas constant	$R = 8.31\ \mathrm{J\,mol^{-1}\,K^{-1}}$
Planck's constant	$h = 6.626 \cdot 10^{-34}\ \mathrm{J\,s}$
Dirac's constant	$\hbar = \frac{h}{2\pi} = 1.055 \cdot 10^{-34}\ \mathrm{J\,s}$
Rydberg's constant	$Ry = 1.097 \cdot 10^{7}\ \mathrm{m^{-1}}$
Bohr atomic radius	$a_B = 5.29 \cdot 10^{-11}\ \mathrm{m}$

2 Mechanics

2.1 Kinetics of point-masses

▶ Position, velocity and acceleration

differentiate → differentiate →

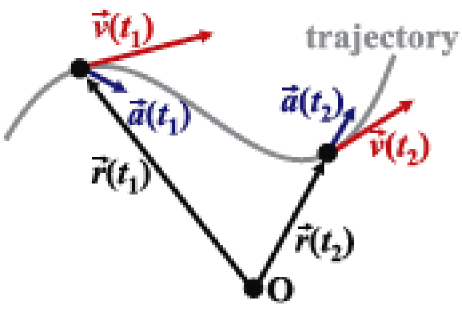
Position	Velocity = change of position per time interval	Acceleration = change of velocity per time interval
$t \mapsto x(t)$ unit: $[x] = \mathrm{m}$	Mean velocity between t_1 and t_2: $\bar{v} = \frac{\Delta x(t)}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$ unit: $[v] = \frac{\mathrm{m}}{\mathrm{s}}$	Mean acceleration between t_1 and t_2: $\bar{a} = \frac{\Delta v(t)}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$ unit: $[a] = \frac{\mathrm{m}}{\mathrm{s}^2}$
Assigns a unique position $x(t)$ to an object for each time t $x(t) = \int v(t)\,\mathrm{d}t$	Momentary velocity at time t: $v(t) = x'(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t}$ $v(t) = \int a(t)\,\mathrm{d}t$	Momentary acceleration at time t: $a(t) = v'(t) = x''(t) = \frac{\mathrm{d}v(t)}{\mathrm{d}t} = \frac{\mathrm{d}^2 x(t)}{\mathrm{d}^2 t}$

← integrate ← integrate

Principle of Superposition: motions in x, y and z - direction are independent of each other (coupled via the time t).

More-dimensional motion:

position: $\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$ velocity: $\vec{v}(t) = \begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix}$ acceleration: $\vec{a}(t) = \begin{pmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{pmatrix}$



▶ Uniform motion and uniformly accelerated motion

	position	velocity	acceleration
Uniform motion	$x(t) = v\,t + x_0$	$v = \frac{\Delta x}{\Delta t} = \text{constant}$	$a = \frac{\Delta v}{\Delta t} = 0$
Uniformly accelerated motion	$x(t) = \frac{1}{2} a\,t^2 + v_0 t + x_0$	$v(t) = a\,t + v_0$	$a = \frac{\Delta v}{\Delta t} = \text{constant}$

Particular equations for the uniformly accelerated motion ($x_0 = 0$):

without t:	without a:
$x = \frac{v^2 - {v_0}^2}{2\,a}$	$x(t) = \frac{1}{2}(v + v_0)\,t$

particularly: $v_0 = 0$ (starting from rest)

$x(t) = \frac{1}{2}\,a\,t^2$; $x = \frac{v^2}{2\,a}$; $x(t) = \frac{1}{2}\,v\,t$

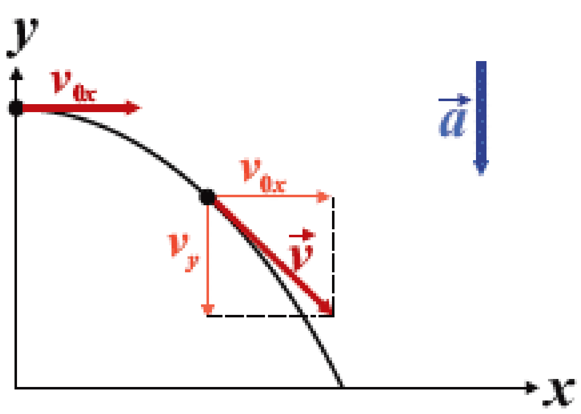
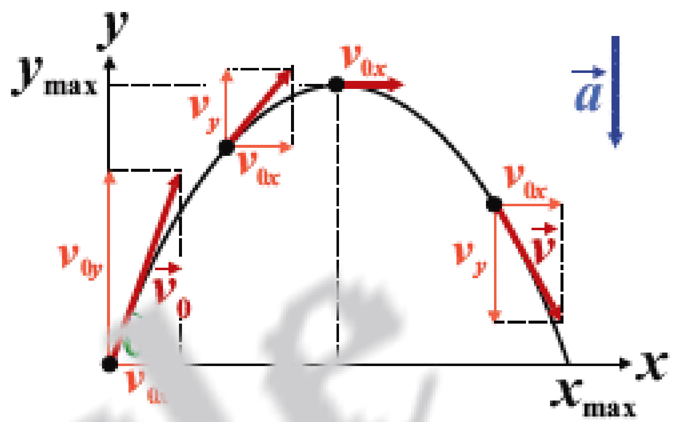
▶ Galileo Galilei

- On Earth's surface, all bodies fall with the same acceleration, provided the air resistance ist neglected. The acceleration of gravity on the Earth's surface is $g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$.
- The position coordinates behave like the squares of the times: $x \propto t^2$, $\quad x(t) = \frac{1}{2}\, g\, t^2$

▶ Projectile motion

Acceleration: $\vec{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$

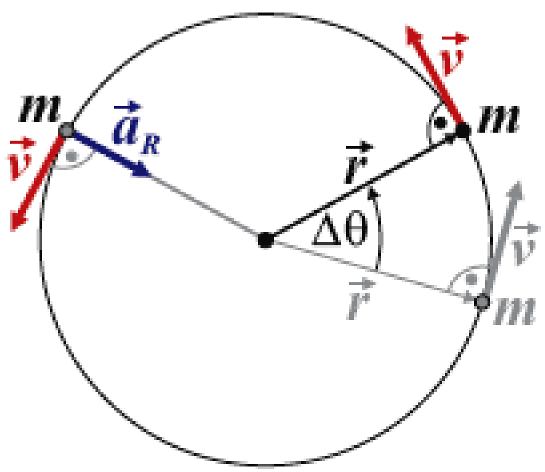
- Uniform motion in x-direction and
- uniformely accelerated motion in y-direction.

horizontal throw		inclined throw
		
$\vec{r}(t) = \begin{pmatrix} v_{0x}\, t \\ -\frac{1}{2}\, g\, t^2 \end{pmatrix}$	**position**	$\vec{r}(t) = \begin{pmatrix} v_0 \cos(\alpha)\, t \\ v_0 \sin(\alpha)\, t - \frac{1}{2} g\, t^2 \end{pmatrix}$
$\vec{v}(t) = \begin{pmatrix} v_{0x} \\ -g\, t \end{pmatrix}$	**velocity**	$\vec{v}(t) = \begin{pmatrix} v_0 \cos(\alpha) \\ v_0 \sin(\alpha) - g\, t \end{pmatrix}$
$y(x) = \frac{-g}{2\, v_{0x}^2}\, x^2$	**trajectory parabola**	$y(x) = \tan(\alpha)\, x - \frac{g}{2\, v_0 \cos^2(\alpha)}\, x^2$

$y_{\max} = \frac{{v_0}^2 \sin^2(\alpha)}{2g}, \quad x_{\max} = \frac{{v_0}^2 \sin(2\alpha)}{g}$

▶ Circular motion: position, velocity and acceleration

- **Period** T = time in s for one revolution.
- **Frequency** f = number of revolutions per time: $f = \frac{1}{T}$
  unit: 1 Hertz = 1 Hz = $\frac{1}{\mathrm{s}}$
- **Angular velocity**
  $\omega = \frac{\text{angle (radians!)}}{\text{time}}$ $\quad \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$
- **Velocity** of a point on the circumference
  $v = \frac{\text{circumference}}{\text{time (period)}}$ $\quad v = \frac{2\pi r}{T} = \omega\, r$
- **Radial (centripetal) acceleration**
  $a_R = \frac{v^2}{r} = r\,\omega^2 \qquad a_R \perp \vec{v},$
  $\Rightarrow a_R$ acts uniquely deflecting, $|v|$ = constant.



2.2 Dynamics of point-masses

▶ Newton's laws

I Principle of inertia:

$\vec{F}_{\text{net}} = \sum \vec{F}_i = \vec{0} \;\Rightarrow\; \vec{v} = \text{constant} \;\Rightarrow\; \vec{a} = \vec{0}$

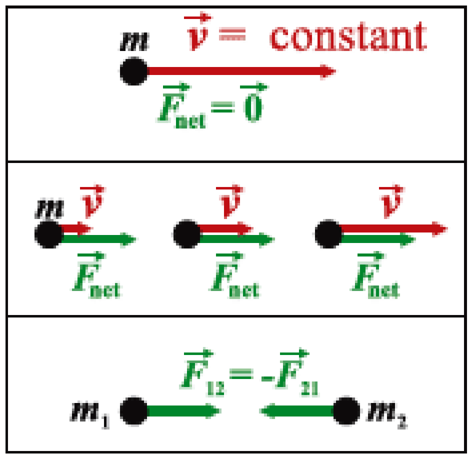
II Principle of action: net force = mass · acceleration

$\vec{F}_{\text{net}} = m \cdot \vec{a}$ unit: $[F] = 1\,\text{Newton} = 1\,\text{N} = 1\,\text{kg}\,\frac{\text{m}}{\text{s}^2}$

III Principle of reaction:

For each acting force there is a counterforce

action = reaction

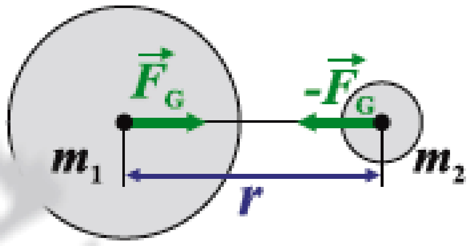


▶ Mechanical forces

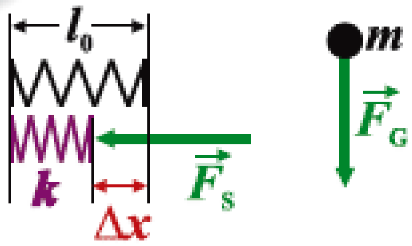
- **Universal force of gravitation:** $F_G = G\,\frac{m_1 \cdot m_2}{r^2}$

  Gravitational constant $G = 6.67 \cdot 10^{-11}\,\frac{\text{N}\,\text{m}^2}{\text{kg}^2}$

  r = distance between the centers of mass.

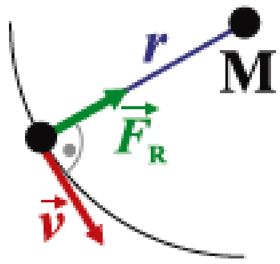


- **Gravitational force:** $F_G = m \cdot g$ where $g \approx 9.81\,\frac{\text{m}}{\text{s}^2}$ is the acceleration of gravity on the Earth's surface. Compare table on p. 34 for other planets.



- **Spring force:** $F_S = k \cdot \Delta x$ where $k = \frac{F}{\Delta x}$ is the spring constant or spring stiffness. Unit: $[k] = \frac{\text{N}}{\text{m}}$

  Springs in series: $\frac{1}{k_{\text{tot}}} = \frac{1}{k_1} + \frac{1}{k_2}$ parallel: $k_{\text{tot}} = k_1 + k_2$



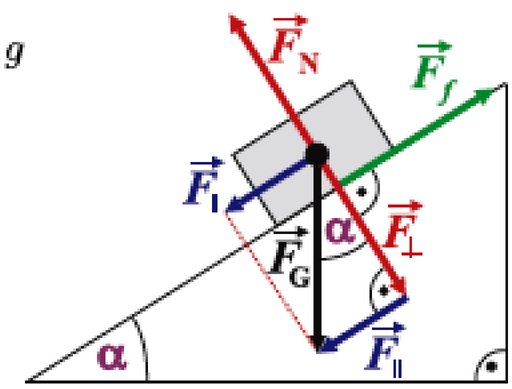
- **Radial force:** $F_R = m \cdot \frac{v^2}{r} = m\,r\,\omega^2 = \frac{4\pi^2 m\,r}{T^2}$

  F_R = force necessary to keep a mass on a circular trajectory.

⇒ Electrical and magnetic forces see p. 20 and 23. Planet masses see p. 34.

▶ Inclined plane, frictional force

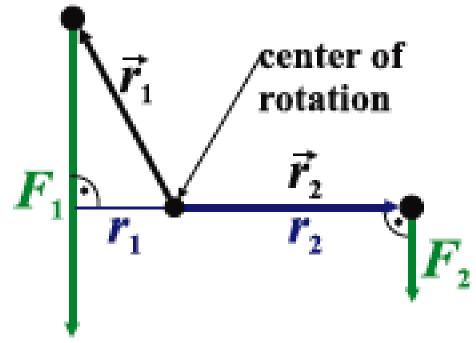
- **Gravitational force:** $\vec{F}_G = \vec{F}_\perp + \vec{F}_\parallel$, with $|\vec{F}_G| = F_G = m \cdot g$
- **Perpendicular component:** $F_\perp = F_G \cdot \cos(\alpha)$
- **Parallel component:** $F_\parallel = F_G \cdot \sin(\alpha)$
- **Frictional force:** $F_{fr} = \mu \cdot F_\perp$



coefficient of friction $\mu = \begin{cases} \mu_S = \textbf{S}\text{tatic friction} \\ \mu_K = \textbf{K}\text{inetic friction} \\ \mu_R = \textbf{R}\text{olling friction} \end{cases}$

$\mu_S > \mu_K > \mu_R$

⇒ see table on p. 31.



▶ Torque $\vec{\tau}$

- **Torque** = force · lever arm, $\tau = F_\perp \cdot r = F \cdot r_\perp$
  general: $\vec{\tau} = \vec{r} \times \vec{F}$ unit: $[\tau] = \mathrm{N \cdot m}$
- **Law of the lever:** $F_1 \cdot r_1 = F_2 \cdot r_2$

▶ Equilibrium (statics)

Sum of all forces $\vec{F}_{\text{net}} = \sum_i \vec{F}_i = \vec{0}$ and sum of all torques $\vec{\tau}_{\text{net}} = \sum_i \vec{\tau}_i = \vec{0}$

▶ Momentum $\vec{p}$

Momentum = mass · velocity, $\vec{p} = m \cdot \vec{v}$ unit: $[p] = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$

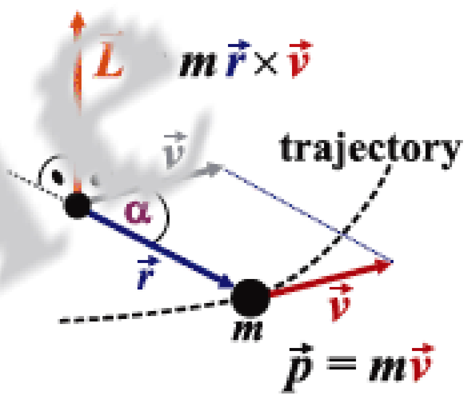
Without external forces, total momentum $\vec{p}_{\text{tot}}$ is conserved: $\vec{F}_{\text{ext}} = \vec{0} \;\Rightarrow\; \vec{p}_{\text{tot}} = \text{constant}$.

▶ Angular momentum $\vec{L}$

$\vec{L} = m\,\vec{r} \times \vec{v}$, $|\vec{L}| = m\,r\,v\,\sin(\alpha)$ unit: $[L] = \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}$

Rigid solids: $L = I \cdot \omega$ rotational inertia I see p. 9.

$\vec{L}$ is a conservational quantity: $\vec{\tau}_{\text{ext}} = \vec{0} \Rightarrow \vec{L} = \text{constant}$.



▶ Law of conservation of angular momentum

Torque = change of angular moment with time:

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t} = \frac{d\vec{L}}{dt} = \vec{L}\,'(t)$$

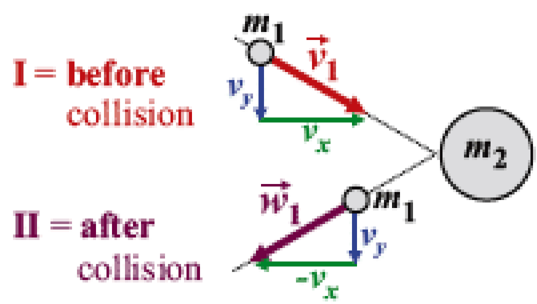
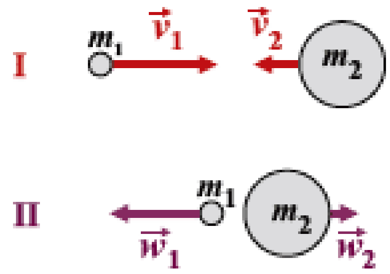
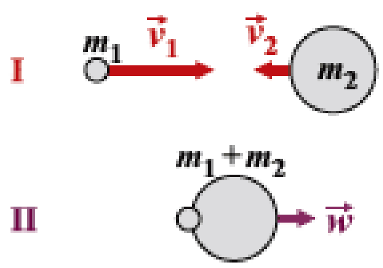
Rigid solids: $\tau = I \cdot \frac{\Delta \omega}{\Delta t} = I \cdot \omega'(t)$ rotational inertia I see p. 9.

▶ Generalisation of $F = m\,a$

Force = change of momentum with time: $\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{d\vec{p}}{dt} = \vec{p}\,'(t)$ or

$\vec{F} = m(t)\,\vec{a} + m'(t)\,\vec{v}$ (product rule) for time-dependent masses (rockets).

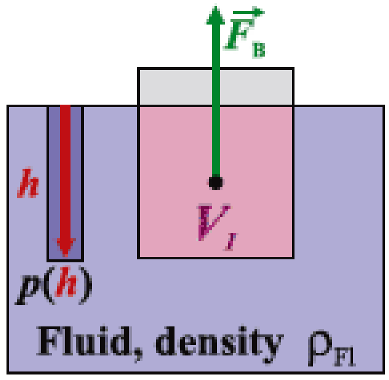
▶ Collisions:

Totally elastic collision	**Partially elastic collision**	**Totally inelastic collision**
		
Momentum: $\vec{p}_{\text{tot}}^{\,\text{I}} = \vec{p}_{\text{tot}}^{\,\text{II}}$	$\vec{p}_{\text{tot}}^{\,\text{I}} = \vec{p}_{\text{tot}}^{\,\text{II}}$	$\vec{p}_{\text{tot}}^{\,\text{I}} = \vec{p}_{\text{tot}}^{\,\text{II}}$
Kin. Energy: $K_{\text{trans}}^{\text{I}} = K_{\text{trans}}^{\text{II}}$	no E-conservation	$K_{\text{trans}}^{\text{I}} - K_{\text{trans}}^{\text{II}} = \frac{m_1\,m_2\,(v_1 - v_2)^2}{2\,(m_1 + m_2)}$

2.3 Hydrostatics, pressure, density

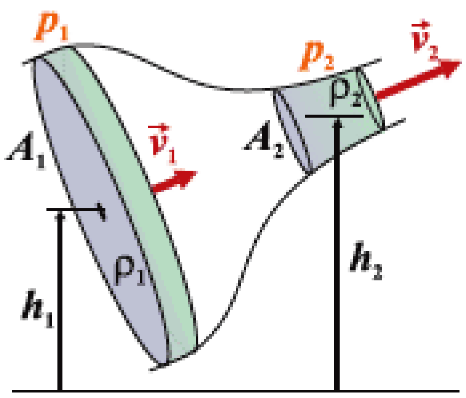
▶ **Density** = mass per volume $\rho = \frac{m}{V}$ unit: $[\rho] = \frac{\text{kg}}{\text{m}^3}$ or $\frac{\text{g}}{\text{cm}^3}$ table p. 30 ff.

▶ **Pressure** = force per area $p = \frac{F}{A}$ unit: $[p] = 1\,\text{Pascal} = 1\,\text{Pa} = 1\,\frac{\text{N}}{\text{m}^2}$,
$1\,\text{bar} = 10^5\,\text{Pa}$



- **Hydrostatic pressure**: pressure of a column of height h:
  $p(h) = \rho_{Fl} \cdot g \cdot h$ (for constant fluid density ρ_{Fl})
- **Buoyant force, Archimede's law:**
  Buoyant force = weight of the suppressed fluid:
  $F_B = \rho_{Fl} \cdot g \cdot V_I$ with V_I = immersed volume and
  ρ_{Fl} = density of the fluid.
- **Barometric formula:** Air pressure in altitude h above sea level: $p(h) = p_0 \cdot e^{-\frac{\rho_0}{p_0} g\, h}$
  with $p_0 = 1.013 \cdot 10^5$ Pa and $\rho_0 = 1.293\,\frac{\text{kg}}{\text{m}^3}$ are the atmospheric pressure resp. density on sea level (at $T_0 = 0\,°\text{C}$) (normal conditions).

2.4 Hydrodynamics

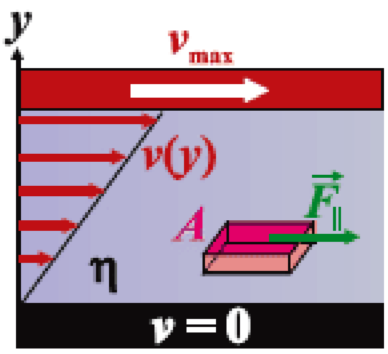


▶ **Continuity equation:** $\rho_1 \cdot A_1 \cdot v_1 = \rho_2 \cdot A_2 \cdot v_2$

In words: (influent mass = effluent mass) per time.

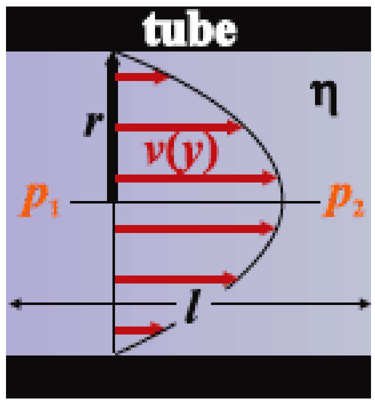
▶ **Bernoulli's law:** $p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$ or

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2$$



▶ **Shear stress and viscosity:** $\tau = \frac{\Delta F_\parallel}{\Delta A} = \eta\,\frac{\Delta v}{\Delta y}$

unit $[\tau] = \frac{\text{N}}{\text{m}^2}$ (compare p. 10)
η = Viscosity, unit: $[\eta] = \frac{\text{N s}}{\text{m}^2}$, table p. 30.

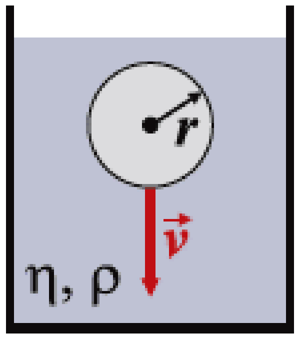


▶ **Hagen Poiseuille's law:** Mean flow velocity $\overline{v}$ in a cylindrical tube of radius r and pressure difference per length $\frac{\Delta p}{l} = \frac{p_2 - p_1}{l}$: $\overline{v} = \frac{\Delta p}{l} \cdot \frac{r^2}{8\eta}$

Volume ΔV flowing across a section per time Δt : $\frac{\Delta V}{\Delta t} = \pi \cdot \frac{\Delta p}{l} \cdot \frac{r^4}{8\,\eta}$

▶ **Stoke's friction force**: For small v, resp. laminar flow (no turbulences)

$F_{fr} = 6\,\pi\,\eta\,r\,v$ η = viscosity see p. 8, table on p. 30.



▶ **Friction force for turbulent flow**: For large v turbulences may occur.

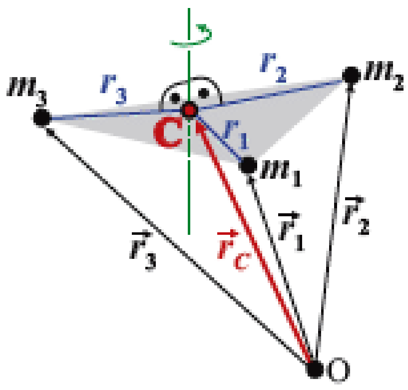
Then: $F_{fr} = \frac{1}{2}\,c_W\,A\,\rho\,v^2$

where A is the cross section area, ρ the density of the fluid and c_W = constant depending on the shape of the front surface.

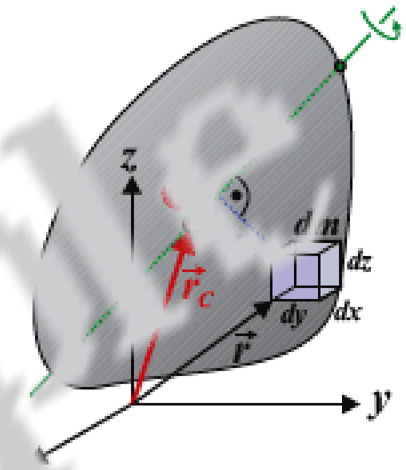
2.5 Mechanics of rigid bodies

▶ **Law of center-of-mass:** A rigid solid behaves like if all external forces act to its center-of-mass C.

n points of mass



solid (continuous mass distribution)



$$\vec r_C = \frac{m_1\,\vec r_1 + m_2\,\vec r_2 + \ldots + m_n\,\vec r_n}{m_1 + m_2 + \ldots + m_n} = \frac{1}{m_{tot}} \sum_{k=1}^{n} m_k\,\vec r_k$$

$$\vec r_C = \frac{1}{m_{tot}} \int \vec r\,dm$$

▶ **Rotational inertia:** The rotational inertia measures "*how much*" mass is located "*how far*" from the rotational axis.

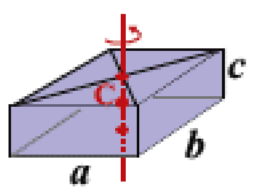
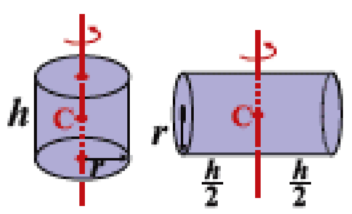
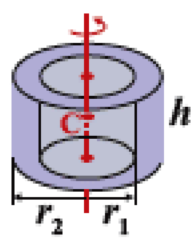
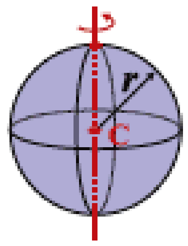
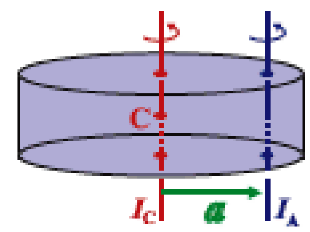
$$I = \sum_{k=1}^{n} m_k \cdot r_k{}^2$$

r_k is the perpendicular distance of mass m_k to the rotational axis.

$$I = \int r_\perp{}^2 dm = \int r_\perp{}^2 \rho\, dV$$

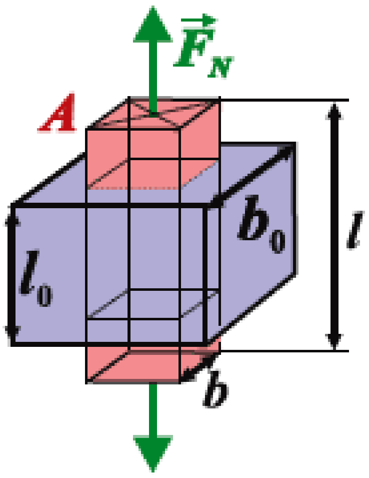
Mass $dm = \rho\, dV = \rho\, dx\, dy\, dz$ at position $\vec r$ at a perpendicular distance $r_\perp$ to the rotational axis.

Rotational inertia I for particular solids:

Cuboid	Cylinder	Hollow cylinder	Sphere	Steiners's law
				
$m(a^2+b^2)$	$\frac{1}{2}mr^2$ $m\left(\frac{r^2}{4}+\frac{h^2}{12}\right)$	$\frac{1}{2}m\,(r_1^2+r_2^2)$	$\frac{2}{5}m\,r^2$	$I_A = I_C + m\,a^2$

▶ Elasticity of rigid solids

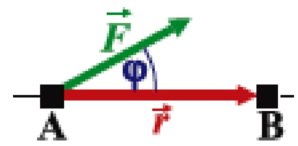
- **Stress** = force per area (as pressure) $\sigma = \frac{\Delta F_N}{\Delta A} = -p$ unit: $[\sigma] = \frac{\text{N}}{\text{m}^2}$



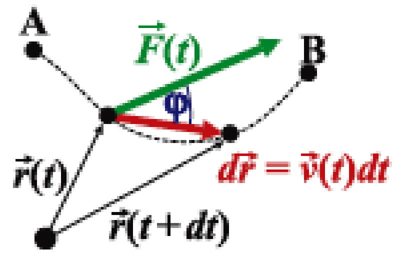
- **Hooke's law:** Deformation $\Delta l = l - l_0$ is proportional to the stress σ: $\frac{\Delta l}{l} = \frac{\sigma}{E} = \varepsilon$ ε = strain.
  E = Young's modulus, unit $[E] = \frac{\text{N}}{\text{m}^2}$, table p. 30.
- **Transversal contraction:** An elongated ($\Delta l > 0$) body will decrease in diameter by $\Delta b = b - b_0 < 0$: $\frac{\Delta b}{b} = -\mu \frac{\Delta l}{l}$
  μ = Poisson's number, table p. 30.
- **Compression:** Compression is proportional to the applied stress:
  $\frac{\Delta V}{V} = \frac{\sigma}{B}$ B = Bulk modulus , unit: $[B] = \frac{\text{N}}{\text{m}^2}$

  Relation between E, μ and B: $B = \frac{E}{3\,(1-2\mu)}$ (homogeneous, isotropic and elastic)
- **Energy density** = energy per volume $w = \frac{\sigma^2}{2E}$

2.6 Work, energy, power



- **Definition:** Work = force · distance
  $W = \vec{F} \cdot \vec{r} = F \cdot r \cdot \cos(\varphi)$ (scalar product)
  unit: $[W]$ = 1 Joule = 1 J = 1 N·m



- **for curvilinear trajectories:** $W_{\text{A}\to\text{B}} = \int_{r_A}^{r_B} \vec{F}(\vec{r})d\vec{r}$
- **Energy:** „stored" work, ability of performing work.

▶ Mechanical work, energy

- **Potential energy:**
  $U_{\text{grav}} = m g\, y$ y = vertical altitude
- **Kinetic energy:**
  $K_{\text{trans}} = \frac{1}{2}\, m\, v^2$ v = velocity
- **Spring energy:**
  $U_{\text{spring}} = \frac{1}{2}\, k\, x^2$ spring stiffness $k = \frac{\Delta F}{\Delta x}$
- **Rotational energy:**
  $K_{\text{rot}} = \frac{1}{2}\, I\, \omega^2$ ω = angular velocity

- **Gravitational energy:** $U_{\text{A}\to\text{B}} = \int_{r_A}^{r_B} F_G(r)\, dr = G\, m_1\, m_2 \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$

  Work, necessary to bring m_1 from position A (r_A) to position B (r_B).

- **Energy conservation:** The total energy of a system is conserved with time:

$$E_{\text{tot}} = \text{constant} \quad \Rightarrow \quad \frac{dE_{\text{tot}}}{dt} = 0.$$

For systems without friction: $E_{\text{tot}} = U_{\text{grav}} + U_{\text{spring}} + K_{\text{trans}} + K_{\text{rot}} = \text{constant}$

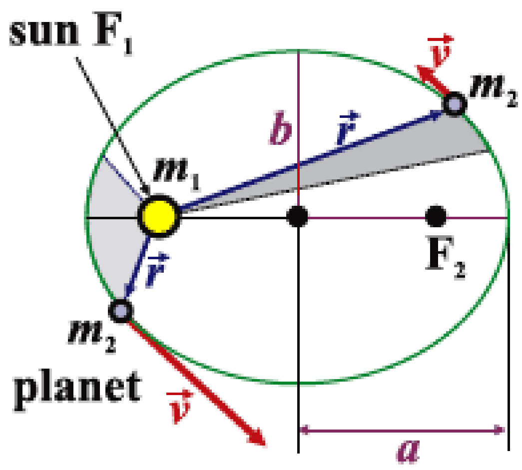
- **Kepler's laws**

I Planets move on elliptical orbits. The sun is located in one of its focal points F_1.

II The area covered by the radius vector $\vec{r}$ per time unit is constant: $\Leftrightarrow \vec{L} = m\,\vec{r} \times \vec{v} = \text{constant}$.

III The ratio of the squares of the periods T of any two planets revolving about the sun is equal to the ratio of the cubes of their semimajor axes a:

$$\frac{T_1^2}{T_2^2} = \frac{a_1{}^3}{a_2{}^3}$$ for circular orbits: $a \approx r$.



⇒ Astronomical data see p. 34.

- **Power**

Definition: Power = work per time $P = \frac{W}{t}$ unit: $[P] = 1$ Watt $= 1\,\text{W} = 1\frac{\text{J}}{\text{s}}$

⇒ Electric power see p. 20.

- **Efficiency**

Definition: Efficiency coefficient $= \frac{\text{usable energy}}{\text{supplied energy}} = \frac{\text{usable power}}{\text{supplied power}}$

$$\eta = \frac{E_{\text{use}}}{E_{\text{in}}} = \frac{P_{\text{use}}}{P_{\text{in}}}$$

2.7 Analogy translation - rotation

Translation		Rotation
$x(t)$	**position / angle**	$\theta(t)$
$v(t)$	**velocity / angular velocity**	$\omega(t)$
$a(t)$	**acceleration / angular acceleration**	$\alpha(t) = \frac{\Delta\omega}{\Delta t}$
m	**mass / inertia**	I
$F(t) = m\,a(t)$	**force / torque**	$\tau(t) = I\,\alpha(t)$
$p(t) = m\,v(t)$	**momentum / angular momentum**	$L(t) = I\,\omega(t)$
$W_{\text{trans}} = F \cdot r \cdot \cos\varphi$	**work**	$W_{\text{rot}} = \tau \cdot \theta \cdot \cos\varphi$
$K_{\text{trans}} = \frac{1}{2} m\,v^2$	**energy**	$K_{\text{rot}} = \frac{1}{2} I\,\omega^2$
$P_{\text{trans}} = F \cdot v \cdot \cos\varphi$	**power**	$P_{\text{rot}} = \tau \cdot \omega \cdot \cos\varphi$

3 Oscillations

Oscillation = temporal periodic process.

▶ **Harmonic oscillation:** Restoring Force F_R is proportional to the elongation y.

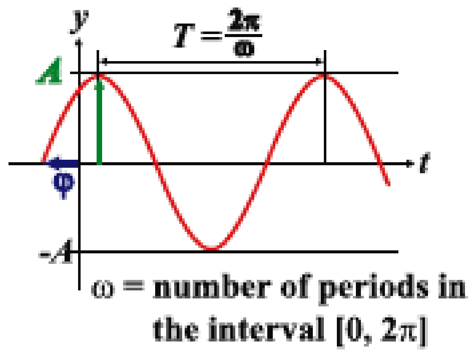
elongation: $y(t) = A \cdot \cos(\omega t + \varphi)$

velocity: $v(t) = y'(t) = -A\omega \sin(\omega t + \varphi)$

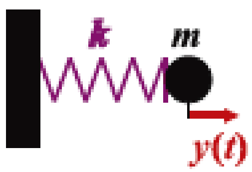
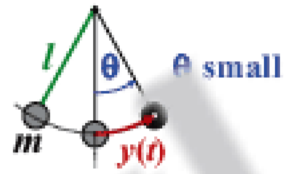
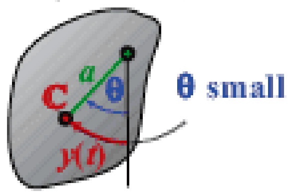
acceleration: $a(t) = y''(t) = -A\omega^2 \cos(\omega t + \varphi) = -\omega^2 y(t)$

A = amplitude (max. elongation)

ω = angular velocity $\omega = \frac{\Delta\varphi}{\Delta t} = \frac{2\pi}{T} = 2\pi f$ φ = phase



▶ **Pendulum: Period** $T = \frac{2\pi}{\omega_0}$

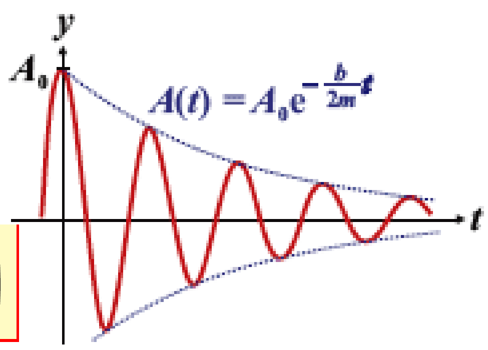
	spring pendulum	gravity pendulum	physical pendulum
			
Angular v.	$\omega_0 = \sqrt{\frac{k}{m}}$	$\omega_0 \approx \sqrt{\frac{g}{l}}$	$\omega_0 \approx \sqrt{\frac{m\,g\,a}{I_C + m\,a^2}}$
Force law	$F_R = -k \cdot y$	$F_R \approx -\frac{m\,g}{l}\,y$	$\tau_R \approx -m\,g\,a\,\theta$
Diff. eq.	$\frac{d^2y}{dx^2} + \frac{k}{m}\,y = 0$	$\frac{d^2y}{dx^2} + \frac{g}{l}\,y \approx 0$	$\frac{d^2\theta}{dx^2} + \frac{m\,g\,a}{I_C + m\,a^2}\,\theta \approx 0$

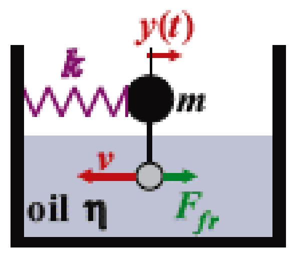
▶ **Damped harmonic oscillation:**

Assume that damping force F_{fr} is proportional to the velocity $v(t)$: $F_{fr} = -b \cdot v$.

With the angular velocity $\omega_0 = \sqrt{\frac{k}{m}}$ of the undamped oscillator, we find:

$$y(t) = A_0\,e^{-\frac{b}{2m}t} \cdot \cos\left(\sqrt{\omega_0^2 - \frac{b^2}{4m^2}} \cdot t + \varphi\right)$$

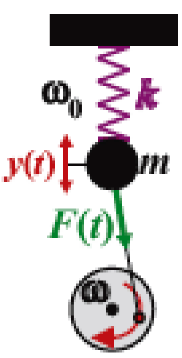




▶ **Forced oscillation, resonance:**

A damped harmonic oscillator is excited by a force $F(t) = F_0 \cdot \cos(\omega t)$. After transcient effects, the elongation can be described by:

$y(t) = A(\omega) \cdot \cos(\omega t + \phi(\omega))$ with:



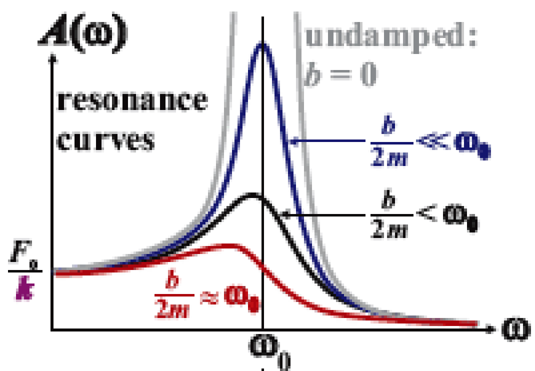
amplitude:

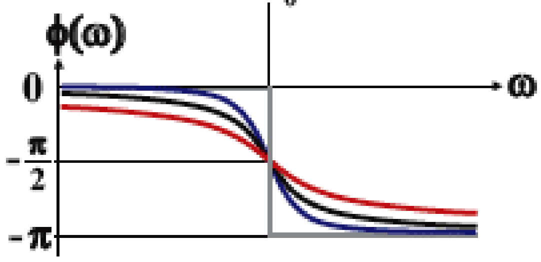
$$A(\omega) = \frac{F_0}{m \cdot \sqrt{(\omega - \omega_0)^2 + \left(\frac{\omega b}{m}\right)^2}}$$

phase:

$$\phi(\omega) = \arctan\left(\frac{\omega_0^2 - \omega^2}{\frac{\omega b}{m}}\right)$$

quality factor: $Q = \frac{m\,\omega_0}{b}$





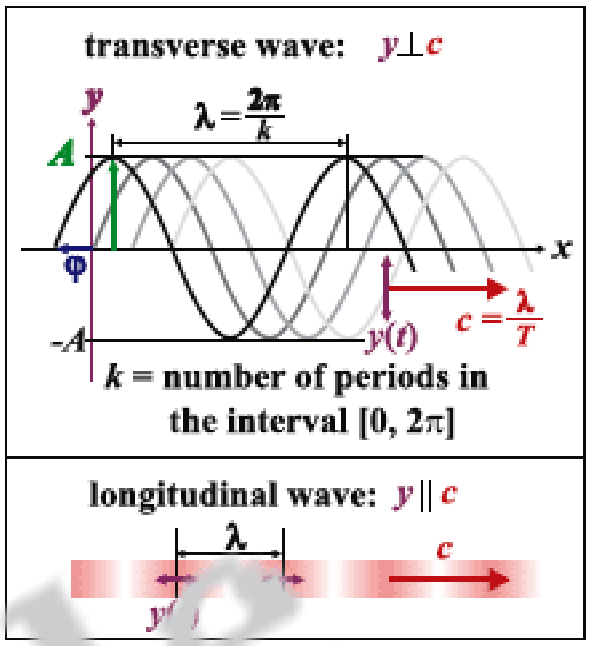
4 Waves

Wave = spatial propagation of an oscillation. Energy is transported, but no mass.

▶ **Harmonic wave:** temporal and spatial periodic process. Wave equation:

$$y(x,t) = A \cdot \sin(\omega\, t \pm k\, x + \varphi)$$
$$y(x,t) = A \cdot \sin\left(2\pi \left(\tfrac{t}{T} \pm \tfrac{x}{\lambda}\right) + \varphi\right)$$

„+" = wave propagating to the left (in $-x$ direction).
„–" = wave propagating to the right (in $+x$ direction).



- **Wave number** $k = \frac{2\pi}{\lambda}$ unit: $[k] = \text{m}^{-1}$

  where λ = wavelength in m (= spatial period)

- **Propagation velocity** $c = \frac{\lambda}{T} = \lambda \cdot f = \frac{\omega}{k}$

▶ **Propagation velocities of waves:** (⇒ table p. [illegible])

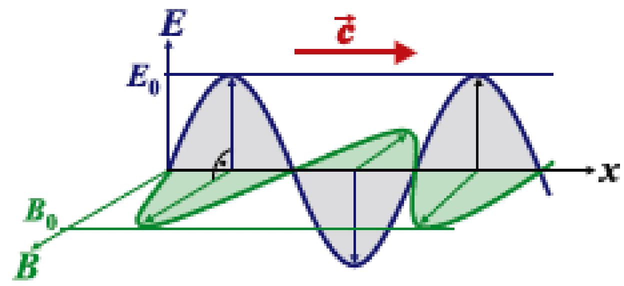
- **Pressure and sound waves**

  gases: $c = \sqrt{\left(\frac{C_p}{C_V}\right)\frac{R\,T}{M}}$ fluids: $c = \sqrt{\frac{B}{\rho}}$ solids: $c = \sqrt{\frac{\sigma}{\rho}}$

  symbols see p. 17 B = bulk modulus, p. 10 σ = tension, p. 10
  air at 20°C: $c = 340\ \frac{\text{m}}{\text{s}}$ ρ = density, p. 8 ρ = density, p. 8

- **Electromagnetic wave:**

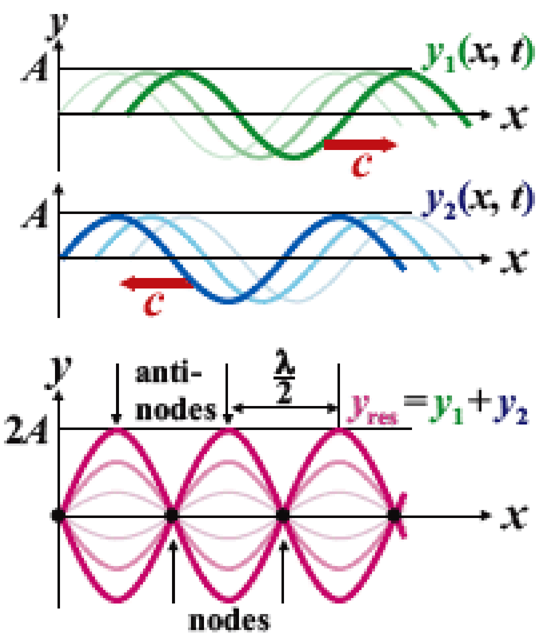
  $c = \sqrt{\frac{1}{\epsilon_0\, \epsilon_r\, \mu_0\, \mu_r}}$ vacuum (air): $\epsilon_r = 1$ and $\mu_r = 1$



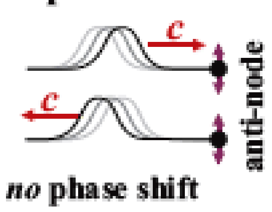
▶ **Standing wave:** Superposition of two identical waves with opposite propagation direction:

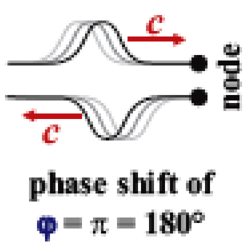
$$y_{\text{res}}(x,t) = A \cdot \sin(\omega\, t - k\, x) + A \cdot \sin(\omega\, t + k\, x)$$

$$y_{\text{res}}(x,t) = \underbrace{2A \cdot \cos(k\, x)}_{A(x)} \cdot \sin(\omega\, t)$$



- **Reflexion of waves:**





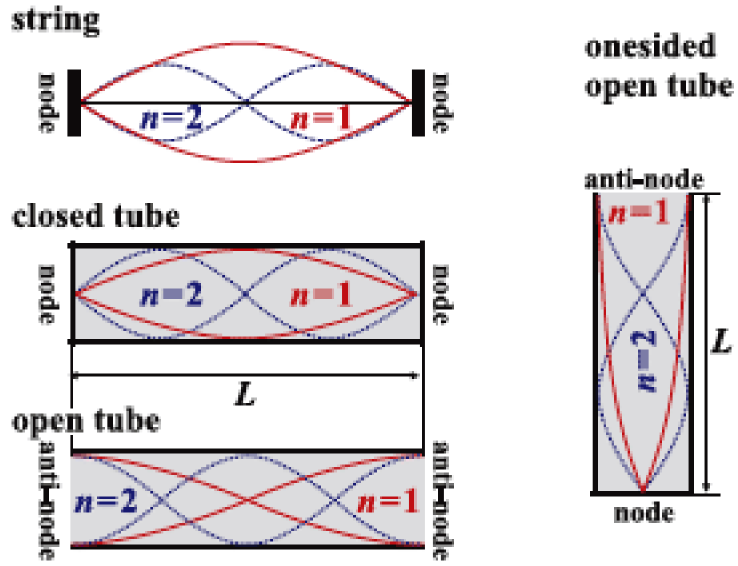
Oscillations
Waves

▶ Eigenfrequencies (condition for standing waves):

- **String:** $f_n = \frac{c}{2L}\, n$ $\quad$ $L = n \cdot \frac{\lambda}{2}$
  order: $n = 1, 2, 3, \dots$
  first harmonic (fundamental): $n = 1$

- **Open or closed tube:**
  $f_n = \frac{c}{2L}\, n$ $\quad$ $L = n \cdot \frac{\lambda}{2}$ $\quad$ $n = 1, 2, 3, \dots$

- **One-sided open tube:**
  $f_n = \frac{c}{4L}\,(2n-1)$ $\quad$ $L = (2n-1) \cdot \frac{\lambda}{4}$

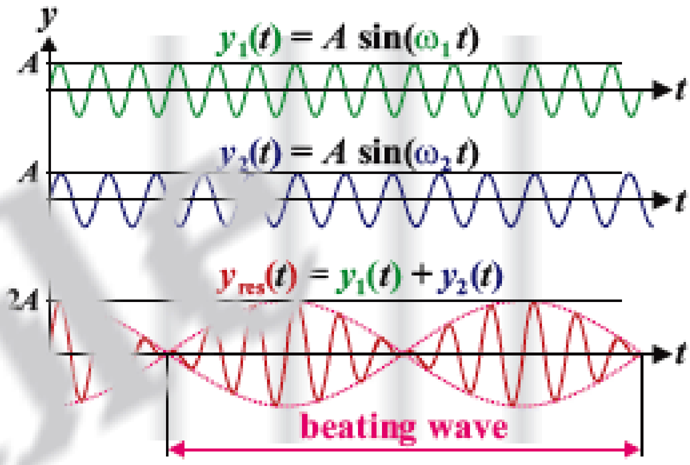


▶ Beats - Interference in time:

Superposition of two oscillations with similar frequencies f_1 and $f_2 = f_1 + \Delta f$.

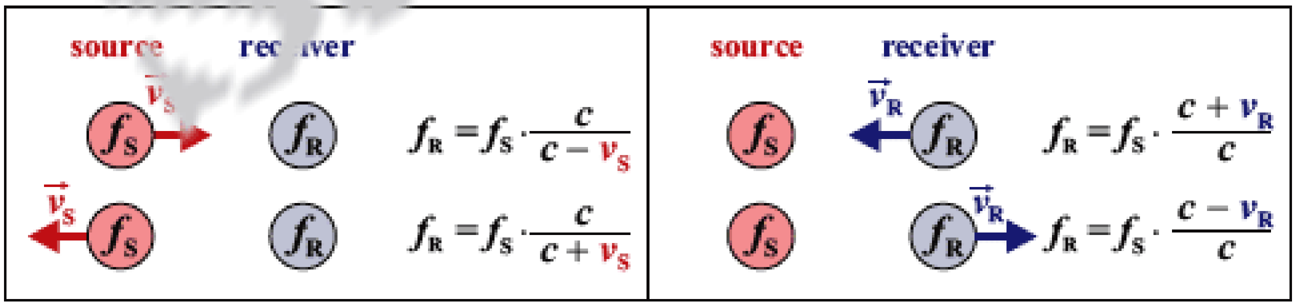
- **Resultant oscillation:**
  $y_{res}(t) = A \cdot \sin(\omega_1\, t) + A \cdot \sin(\omega_2\, t)$
  $$y_{res}(t) = 2A \cdot \sin\left(\left(\omega_1 + \frac{\Delta\omega}{2}\right) t\right) \cdot \cos\left(\frac{\Delta\omega}{2}\, t\right)$$

- **Frequency of beating wave** (audible change in sound intensity): $f_{\text{Beat}} = |f_1 - f_2|$



▶ Doppler effect:

v_S = velocity of the source, v_R = velocity of the receiver. Then: $f_R = \left(\frac{c \pm v_R}{c \mp v_S}\right) \cdot f_S$



▶ Sound intensity:

For an acoustic source of power P emitting a sperical wave, the intensity $I(r)$ depends on the distance r as following: $I(r) = \frac{P}{A} = \frac{P}{4\pi r^2}$ $\quad$ unit: $[I] = \frac{\text{W}}{\text{m}^2}$

sound level: $L = 10 \cdot \log\left(\frac{I}{I_0}\right)$ $\quad$ unit: dB (= Decibel) Audible limit: $I_0 = 10^{-12}\, \frac{\text{W}}{\text{m}^2}$.

5 Optics

Light can be considered either as particles (photons) or as electromagnetic wave. In vacuum (air) the **velocity of light** is constant at $c_0 = 3 \cdot 10^8 \,\frac{\mathrm{m}}{\mathrm{s}}$.

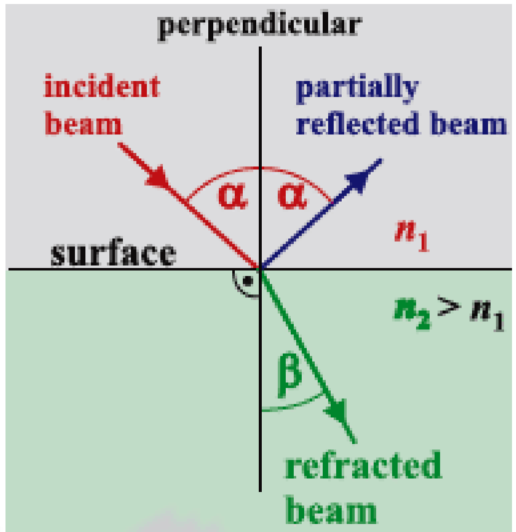
▶ **Index of refraction:** $n = \frac{c_0}{c_M}$, whereas c_M is the velocity of light in the material M.

▶ **Law of refraction(Snellius):** $\frac{\sin(\alpha)}{\sin(\beta)} = \frac{n_2}{n_1} = \frac{c_1}{c_2}$

Indices of refraction see p. 33.

- **total reflection**: $\beta = \beta_{max} = 90°$
- **Brewster angle**: $\tan(\alpha_B) = \frac{n_2}{n_1}$

  Reflected and refracted beam are perpendicular.
  Then, the light of both beams is polarized.



▶ **Imaging with lenses:**

- **Focal length** f: $\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

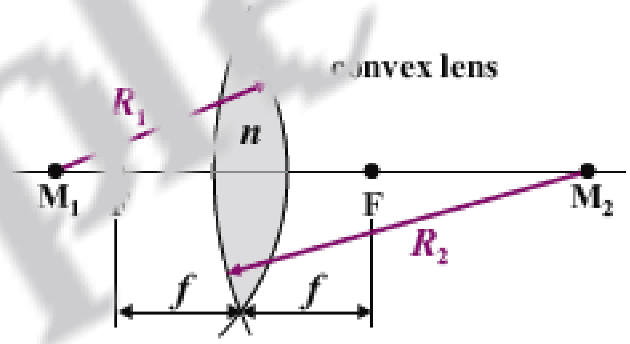
  convex lens: $f > 0$

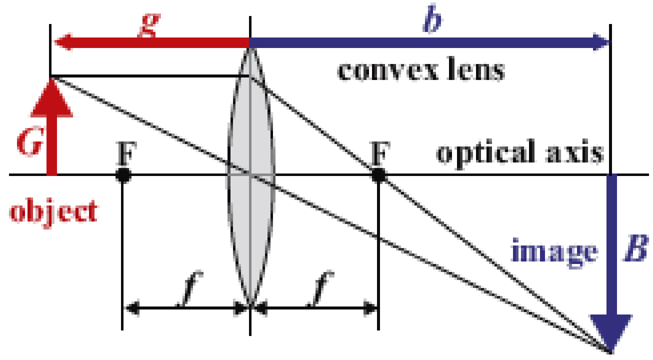
  concave lens: $f < 0$

- Concave mirror: $f \approx \frac{R}{2}$

- Parallel beams of light travelling parallel to the lens axis intersect in the focal point **F**.

- Beams crossing the center of the lens are not deflected.

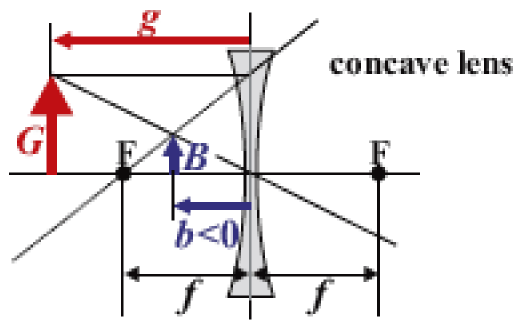




- **Lens equation**: $\frac{1}{f} = \frac{1}{b} + \frac{1}{g}$

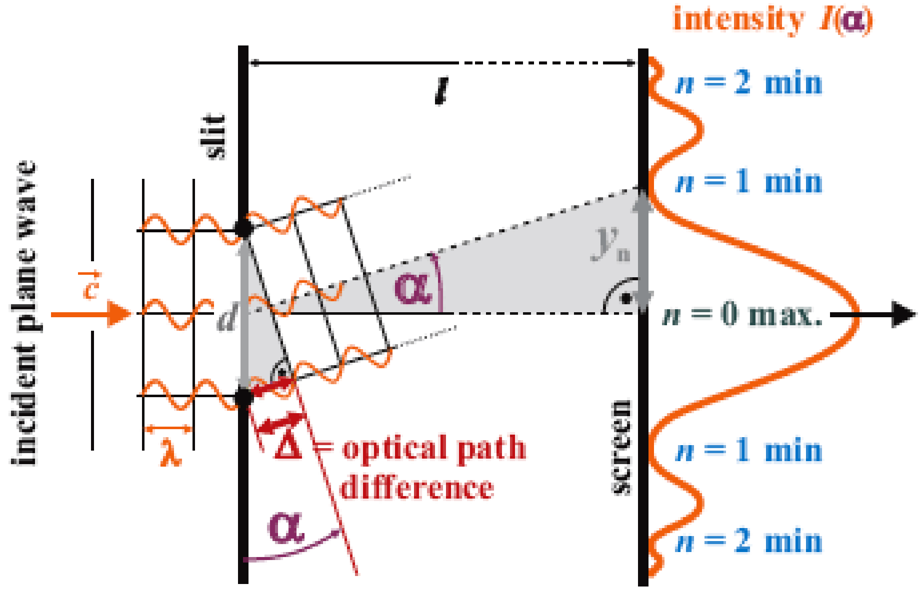
  real image: $b > 0$
  virtual image: $b < 0$

- **Magnification**: $M = \frac{B}{G} = \frac{b}{g}$



▶ **Diffraction:** Deflection of waves (light, water or sound waves) on an obstacle.

- **Diffraction on a single slit:**



Condition for minimal intensity:

$$\Delta = d \cdot \sin(\alpha) = (2n-1) \cdot \lambda$$

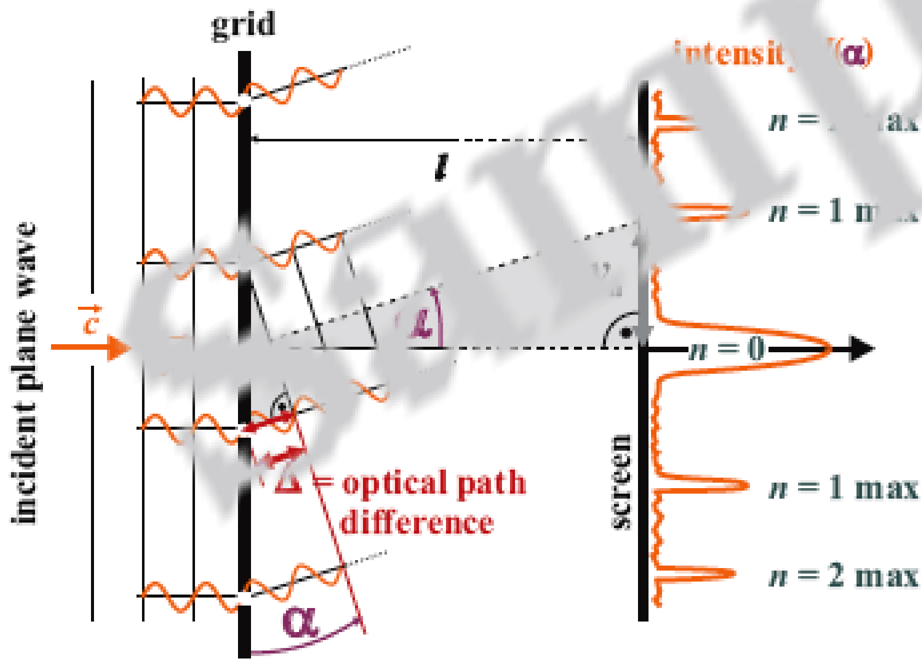
Order: $n = 1, 2, ...$

intensity distribution:

$$I(\alpha) = I_0 \cdot \frac{\sin^2\left(\frac{\phi}{2}\right)}{\left(\frac{\phi}{2}\right)^2} \quad \text{with}$$

$$\phi(\alpha) = \frac{2\pi}{\lambda} \cdot d \sin(\alpha)$$

- **Diffraction on a grid resp. double slit:**



Condition for maximal intensity:

$$\Delta = d \cdot \sin(\alpha) = n \cdot \lambda$$

Order: $n = 0, 1, 2, \ldots$

6 Thermodynamics

▶ **Temperature T:** = Measure for the mean kinetic energy K_{trans} of the particles.
Unit: $[T] = 1\,\text{Kelvin} = 1\,\text{K}$.

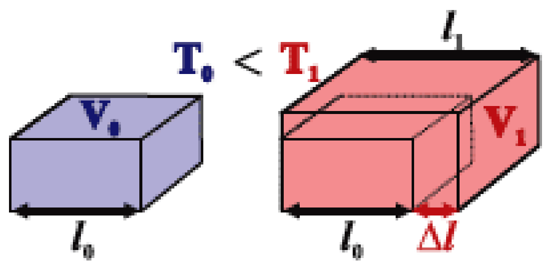
Celsius-Temperature $\boxed{\vartheta = T - 273.15\,\text{K}}$ temperature difference: $\Delta\vartheta = \Delta T = T_1 - T_0$

▶ **Linear expansion:** $\boxed{\Delta l = l_1 - l_0 = \alpha\, l_0\, \Delta T}$

α = coefficient of thermal expansion: table p. 31.

▶ **Volume expansion:** $\boxed{\Delta V = V_1 - V_0 = \gamma\, V_0\, \Delta T}$

$\gamma \cong 3\alpha$ coefficient of volume expansion: table p. 31.



▶ **Heat:** Q (thermal energy)

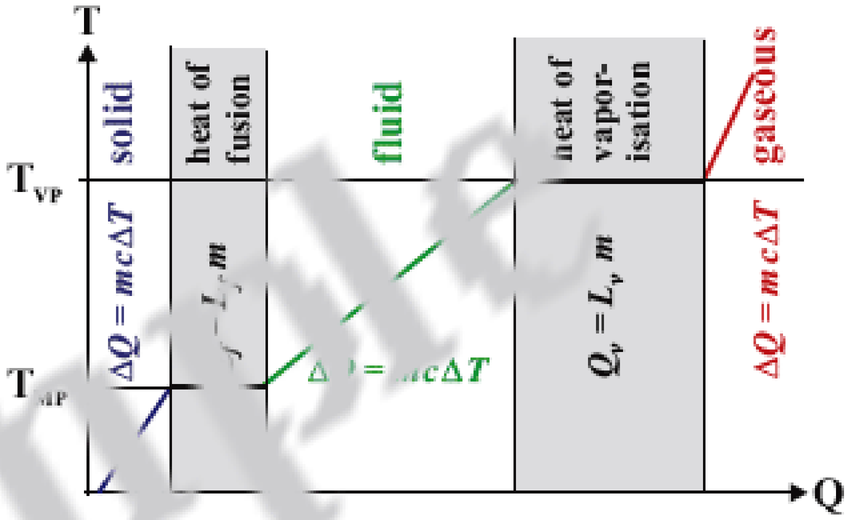
unit: $[Q] = 1\,\text{Joule} = 1\,\text{J}$.

▶ **Heat transfer:** $\boxed{\Delta Q = m\, c\, \Delta T}$

c = specific heat capacity,

unit $[c] = \frac{\text{J}}{\text{kg K}}$ table p. 31.

m = mass in kg.



▶ **Latent heat of fusion:** $\boxed{Q_f = L_f \cdot m}$

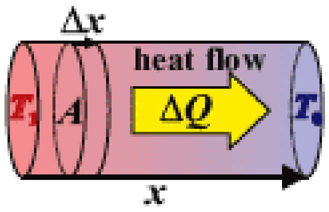
L_f = specific latent heat of fusion, unit: $[L_f] = \frac{\text{J}}{\text{kg}}$ table p. 31.

▶ **Latent heat of vaporisation:** $\boxed{Q_v = L_v \cdot m}$

L_v = specific latent heat of vaporisation, unit: $[L_v] = \frac{\text{J}}{\text{kg}}$ table p. 31.

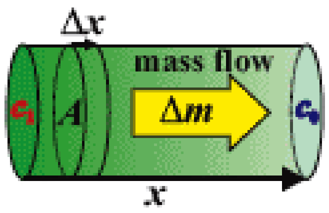
▶ **Thermal conduction:** $\boxed{\frac{\Delta Q}{\Delta t} = -k\, A\, \frac{\Delta T}{\Delta x}}$

k = thermal conductivity, unit: $[k] = \frac{\text{W}}{\text{m K}}$ table p. 31



▶ **Diffusion:** $\boxed{\frac{\Delta m}{\Delta t} = -D\, A\, \frac{\Delta c}{\Delta x}}$ (Fick's law)

D = diffusion constant, unit: $[D] = \frac{\text{m}^2}{\text{s}}$ c = concentration



▶ **Mean kinetic energy:** $\boxed{\overline{K_{\text{trans}}} = \frac{f}{2}\, k_B\, T = \frac{1}{2}\, m\, v_{\text{rms}}^2}$ with $v_{\text{rms}} = \sqrt{f \cdot \frac{k_B T}{m}}$

$\boxed{k_B = 1.38 \cdot 10^{-23}\, \frac{\text{J}}{\text{K}}}$ = Boltzmann's constant.

f = number of degrees of freedom: $\begin{cases} \text{monoatomic particles:} & f = 3 \quad \text{(3 trans.)} \\ \text{diatomic particles:} & f = 5 \quad \text{(3 trans. \& 2 rot.)} \end{cases}$

- **Stefan Boltzmann law:** Radiant heat flux: $\frac{\Delta Q}{\Delta t} = \varepsilon \cdot \sigma \cdot A \cdot T^4$ with
  ε = emissivity, $\varepsilon = 1$ for perfect blackbody
  $\sigma = 5.67 \cdot 10^{-8} \frac{\mathrm{J}}{\mathrm{s\,m^2\,K^4}}$ = Stefan-Boltzmann constant
  A = surface area of emitting body of temperature T.

- **Entropy:** = "Measure of the disorder of a system"   unit: $[S] = \frac{\mathrm{J}}{\mathrm{K}}$
  - **Thermodynamic:** $\Delta S = \frac{\Delta Q}{T}$
  - **Statistic:** $S = k_B \ln(P)$ P = number of states at constant total energy and constant number of particles.

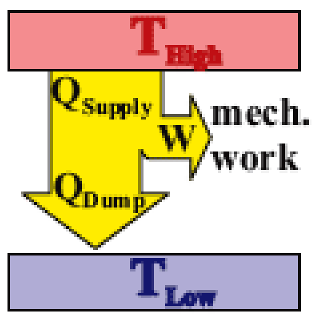
1. Law of thermodynamics (energy conservation):

The change in the internal energy of a closed thermodynamic system is equal to the sum of the amount of heat energy supplied and the work done on the system: $\Delta U = \Delta Q + \Delta W$

Pressure-volume work: $\Delta W = -p\,\Delta V$ if p = constant. General: $\Delta W_{12} = -\int_{V_1}^{V_2} p(V)\,dV$

2. Law of thermodynamics (heat engines):

- Heat will never spontaneously flow from a colder to a warmer body.
- Heat cannot be completely converted into mechanical work:
  $\Delta W = Q_{\text{Supply}} - Q_{\text{Dump}}$
- The maximum possible thermal efficiency of a heat engine is called Carnot cycle efficiency $\eta_C = 1 - \frac{T_{\text{Low}}}{T_{\text{High}}} = 1 - \frac{|Q_{\text{Dump}}|}{|Q_{\text{Supply}}|}$



- **Thermal efficiency:** $\eta = \frac{W_{\text{Useful}}}{Q_{\text{Supply}}} = \frac{W_{\text{Useful}}}{W_{\text{Supply}}} = \frac{P_{\text{Useful}}}{P_{\text{Supply}}}$ (see also p. 11)

- **Ideal gas law:**

$$\frac{p\,V}{T} = n\,R = \text{constant} \Leftrightarrow \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

particularly:

T = const	V = const	p = const
$p_1 V_1 = p_2 V_2$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$
(Boyle-Mariotte)	(Amontons)	(Gay-Lussac)

- p = pressure in Pa
- V = volume in $\mathrm{m^3}$
- T = temperature in K
- **Universal gas constant:**
  $R = k_B \cdot N_A = 8.31 \frac{\mathrm{J}}{\mathrm{mol\,K}}$ see p. 3
- $n = \frac{m}{M}$ = number of mol
  m = mass in kg
  M = molar mass in $\frac{\mathrm{kg}}{\mathrm{mol}}$

- **Van der Waals gas (real gas):** $\left(p + \frac{a n^2}{V^2}\right)(V - b n) = n R T$

  a, b = Van der Waals-constants, table p. 32.

- **Adiabatic processes:** No heat exchange: $\Delta Q = 0$.

  With C_p = molar heat capacity of a gas at constant pressure and C_V = heat capacity at constant volume we have:

  $C_p - C_V = R$ and $\kappa = \frac{C_p}{C_V}$, $\kappa \approx 1.4$ for air. See table p. 32.

  - **Isobaric**: p = constant in the $V - T$ chart. Adiabatic equation: $T V^{\kappa - 1} = \text{const.}$
  - **Isothermal**: T = constant in the $p - V$ chart. Adiabatic equation: $p V^{\kappa} = \text{const.}$
  - **Isochoric**: V = constant in the $p - T$ chart. Adiabatic equation: $T^{\kappa} p^{1-\kappa} = \text{const.}$

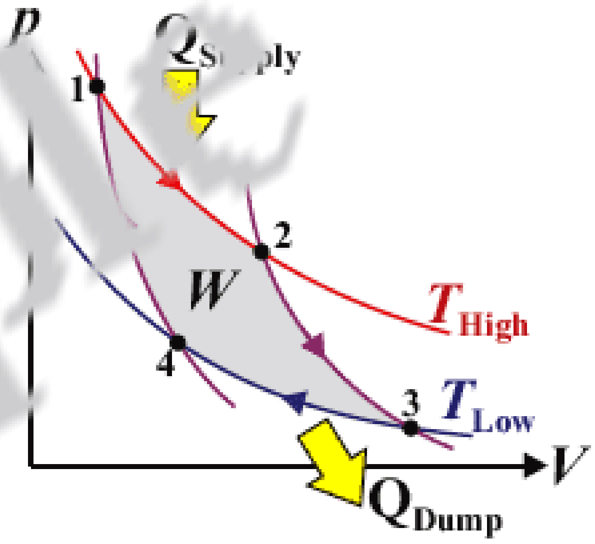
- **Carnot process:**

$1 \to 2$: Isothermal expansion: $Q_{\text{Supply}} = R\,T_{\text{High}} \ln\left(\frac{V_2}{V_1}\right)$

$2 \to 3$: Adiabatic expansion: $\Delta U = C_V\,(T_{\text{Low}} - T_{\text{High}})$

$3 \to 4$: Isothermal compression: $Q_{\text{Dump}} = R\,T_{\text{Low}} \ln\left(\frac{V_4}{V_3}\right)$

$4 \to 1$: Adiabatic compression: $\Delta U = C_V\,(T_{\text{High}} - T_{\text{Low}})$



The maximum possible thermal efficiency of a heat engine is called

Carnot cycle efficiency $\eta_C = 1 - \frac{T_{\text{Low}}}{T_{\text{High}}} = 1 - \frac{|Q_{\text{Dump}}|}{|Q_{\text{Supply}}|}$

7 Electromagnetism

7.1 Electricity

▶ **Electric charge:** $Q = (N_+ - N_-) \cdot e$ with $\left\{ \begin{array}{l} N_+ = \text{number of positive} \\ N_- = \text{number of negative} \end{array} \right\}$ charges.

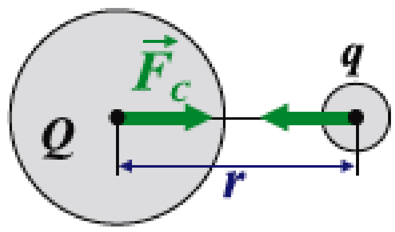
- Unit: $[Q] = 1\,\text{Coulomb} = 1\,\text{C} = \{\text{charge of } 6.25 \cdot 10^{18} \text{ electrons}\}$.
- **Elementary charge:** $e = 1.602 \cdot 10^{-19}\,\text{C}$ **Charge of one electron:** $q = -e$
- In solid conductors, only electrons are mobile.

▶ **Coulomb's law:**

Force F_C between two charges q and Q at distance r:

$$F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{|q \cdot Q|}{r^2} \qquad \epsilon_0 = 8.854 \cdot 10^{-12}\,\frac{\text{C}}{\text{Vm}} = \text{electric constant.}$$

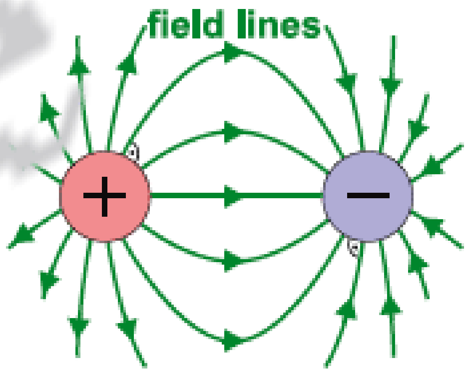
like charges repel, unlike charges attract.



▶ **Electric field E:**

electric field = force per charge: $E = \frac{F_C}{q}$

Electric charges are the source of an electric field. Electric field lines never intersect and are always perpendicular to the conductor's surface.



▶ **Voltage, electric potential V:**

Work W performed by the charge q:

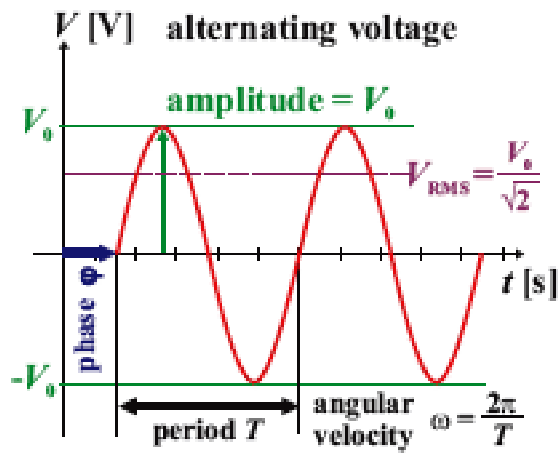
voltage = work per charge: $V = \frac{W}{q}$ unit: $[V] = 1\,\text{Volt} = 1\,\text{V} = 1\frac{\text{J}}{\text{C}}$

- **Direct voltage:** $V = V_0 = \text{constant}$ (battery).
- **Alternating voltage:** $V(t) = V_0 \cdot \sin(\omega \cdot t - \varphi)$ (line voltage).

  V_0 = amplitude, $\omega = \frac{2\pi}{T}$ = angular velocity,

  φ = phase (see p. 12).

  rms-voltage: $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ = value of the equivalent direct voltage, that produces the same power dissipation.

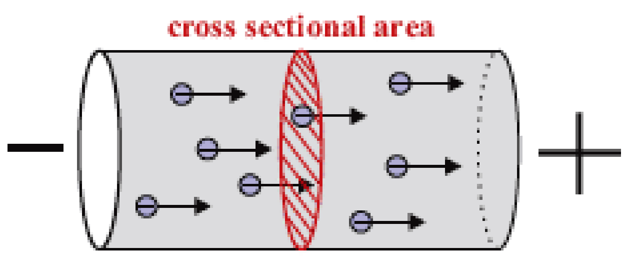


▶ **Current I:**

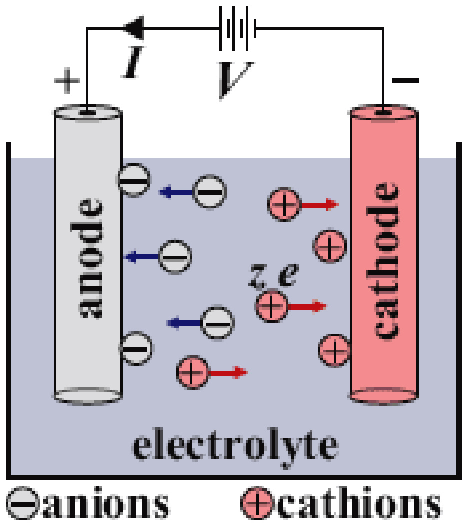
Charge ΔQ per time Δt flowing through the cross sectional area of an electrical conductor:

current = charge per time: $I = \frac{\Delta Q}{\Delta t}$ unit: $[I] = 1\,\text{Ampere} = 1\,\text{A} = 1\frac{\text{C}}{\text{s}}$

- **conventional direction of current:** $\oplus \rightarrow \ominus$.
- **physical direction of current:** electrons flow from $\ominus \rightarrow \oplus$.
- DC = direct current, AC = alternating current.



▶ Electrolysis, ionic current:



Mass m deposited on an electrode due to an ionic current in an electrolyte:

$$m = \frac{M \cdot Q}{z \cdot N_A \cdot e}$$

M = molar mass in $\frac{\text{kg}}{\text{mol}}$

$Q = I \cdot t$ = total charge in C

z = ionic valence

$N_A = 6.022 \cdot 10^{23}\,\text{mol}^{-1}$ = Avogadro's const.

$e = 1.602 \cdot 10^{-19}\,\text{C}$ = elementary charge

▶ Resistance R, Ohm's law:

Resistance = voltage per current: $R = \frac{V}{I}$ unit: $[R] = 1\,\text{Ohm} = 1\,\Omega = 1\frac{\text{V}}{\text{A}}$

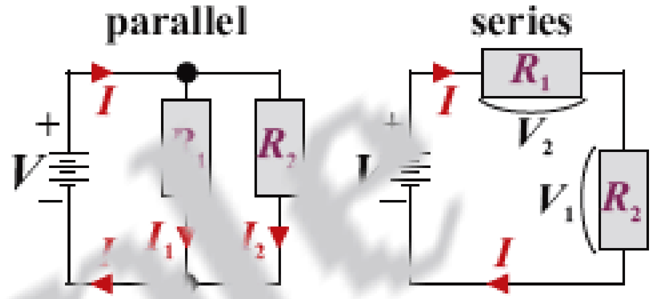
Generalised definition: $R = \frac{\Delta V}{\Delta I} = \frac{dV}{dI}$ = slope in the V-I-chart.

- **Parallel resistors:**

$$R_{\text{tot}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}$$

- **Serial resistors:**

$$R_{\text{tot}} = R_1 + R_2 + \ldots$$

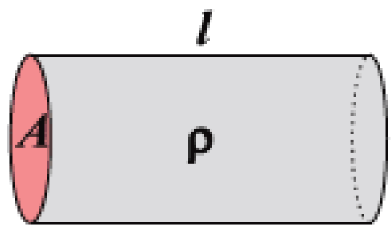


- **Kirchhoff's rules:**

junction rule: $I = I_1 + I_2$

loop rule: $V = V_1 + V_2$

▶ Specific resistance ρ:



Resistance R of a wire of length l [in m] and cross-section A [in m²]: $R = \rho\,\frac{l}{A}$

ρ = **specific resistance** = matter constant (table p. 32).

▶ Electric power P and work W:

power = voltage times current: $P = V \cdot I$ unit: $[P] = 1\,\text{Watt} = 1\,\text{W} = 1\text{V} \cdot \text{A}$

- Other formulae: $P = R \cdot I^2 = \frac{V^2}{R}$ (see also p. 11)

- Active power: $P_A = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos(\varphi)$ where $\varphi = \varphi_2 - \varphi_1$ = phase shift between $V(t)$ and $I(t)$.

Work = power times time: $W = V \cdot I \cdot t$ unit: $[W] = 1\,\text{Joule} = 1\,\text{W s}$
$1\,\text{kWh} = 3.6 \cdot 10^6\,\text{J}$

⇒ Efficiency see p. 11, 18

⇒ Reactance, Impedance, AC and DC-Circuits see p. 25.

Electricity

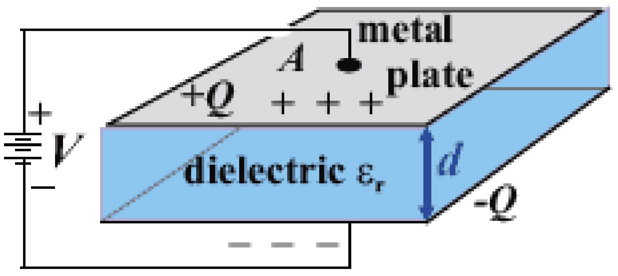
remove this part, please

▶ Capacity C, Capacitors (charge accumulators):

Capacity = charge per voltage: $C = \frac{Q}{V}$ unit: $[C] = 1\,\text{Farad} = 1\,\text{F} = 1\frac{\text{C}}{\text{V}}$.

- **parallel-plate capacitor:** $C = \epsilon_0\,\epsilon_r \cdot \frac{A}{d}$

  $\epsilon_0 = 8.85 \cdot 10^{-12}\,\frac{\text{C}}{\text{Vm}}$
  ϵ_r = electric constant (table p. 33.)
  $\epsilon_r = 1$ for air (vacuum).

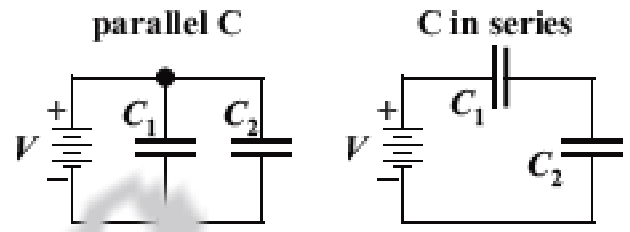


- Electric field in a plate capacitor: $E = \frac{V}{d}$

- **Energy stored in a capacitor:** $E = \frac{1}{2}\,C \cdot V^2$

- **Capacitors in series:** $C_{\text{tot}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right)^{-1}$

  (same charge Q on all capacitors)



- **Capacitors in parallel:** $C_{\text{tot}} = C_1 + C_2 + \ldots$

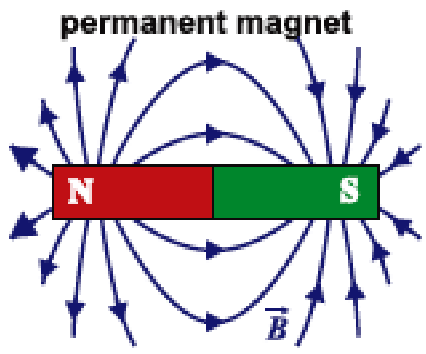
  (same voltage V on all capacitors)

⇒ Reactance, Impedance, AC and DC-Circuits see p. [illegible].

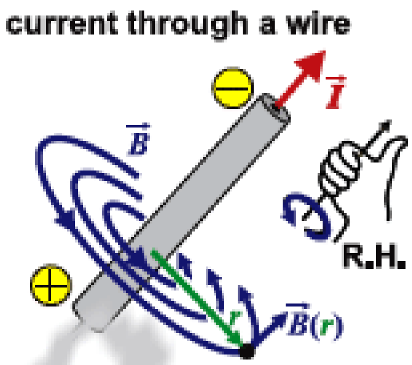
7.2 Magnetism

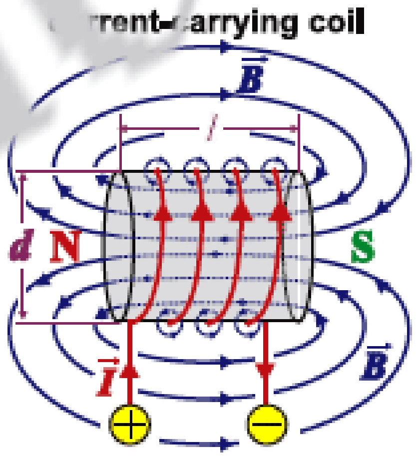
▶ Magnetic filed $\vec{B}$:

A magnetic field causes a force on charges in motion and on magnetic matter (mainly on ferromagnetic elements Fe, Co and Ni). Unit: $[\vec{B}] = 1\,\text{Tesla} = 1\,\text{T} = 1\frac{\text{Vs}}{\text{m}}$



- Magnetic field lines are always closed.
- There are no magnetic monopoles.
- Source of a magnetic field are currents (charges in motion). In the case of permanent magnets these are microscopic circular currents within the material.
- Magnetic field at distance r of a wire with current I: $B(r) = \mu_0\,\mu_r \cdot \frac{I}{2\pi r}$
- Permeability: $\mu_0 = 4\pi \cdot 10^{-7}\,\frac{\text{Vs}}{\text{Am}}$
  μ_r = relative permeability, table p. 32.
  $\mu_r = 1$ for vaccum (air).
- Magnetic field in the center of a circular current I of radius r: $B(r) = \mu_0\,\mu_r \cdot \frac{I}{2\,r}$
- Magnetic field in a coil of N loops of length l and diameter d: $B = \mu_0\,\mu_r \cdot \frac{N \cdot I}{\sqrt{l^2 + d^2}}$



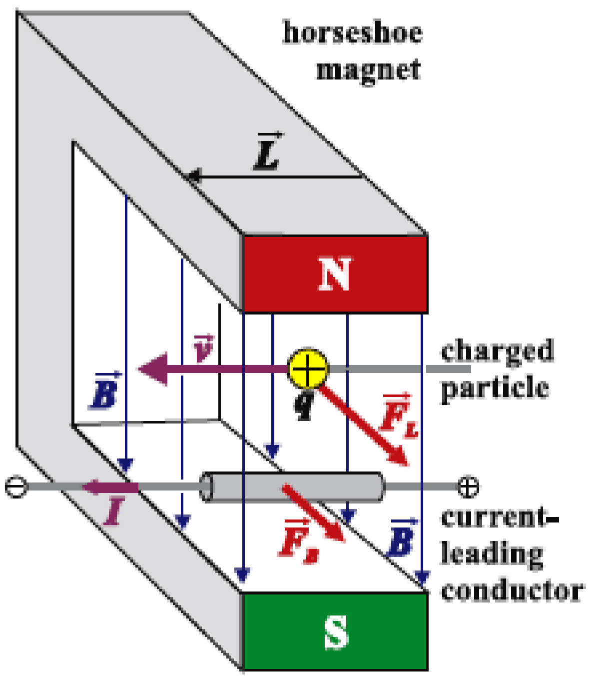


▶ Lorentz force: Force $\vec{F}_L$ acting on a charge q in motion with velocity $\vec{v}$ in a $\vec{B}$-field: $\vec{F}_L = q \cdot \vec{v} \times \vec{B}$

- $\vec{F}_L \perp \vec{B}$ and $\vec{F}_L \perp \vec{v}$
- Magnitude: $F_L = q \cdot v \cdot B \cdot \sin(\alpha)$ $\alpha = \sphericalangle(\vec{v}, \vec{B})$.
- **Lorentz-equation:** $\vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B})$

▶ Biot-Savart force: Force $\vec{F}_B$ acting on a current leading wire in a $\vec{B}$-field: $\vec{F}_B = I \cdot \vec{L} \times \vec{B}$

- $\vec{F}_B$ acts perpendicular to the direction of current $\vec{L}$ as well as perpendicular to the magnetic field $\vec{B}$.
- Magnitude: $F_B = I \cdot L \cdot B \cdot \sin(\alpha)$ $\alpha = \sphericalangle(\vec{L}, \vec{B})$.

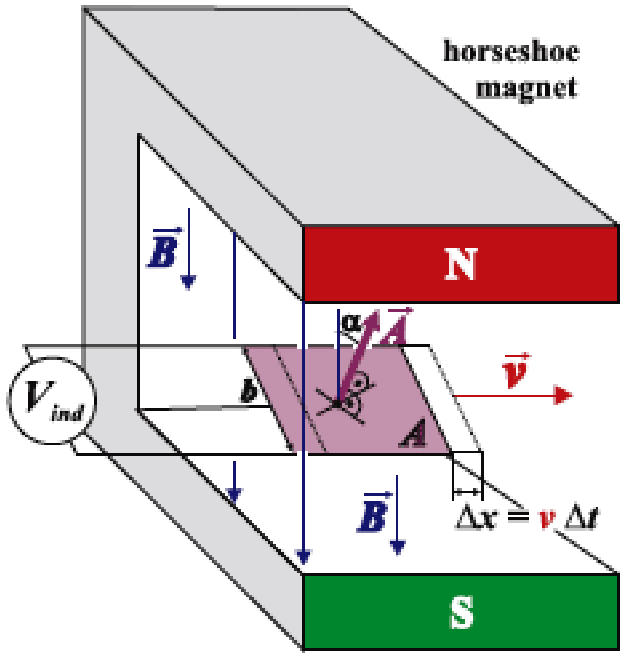


- ▶ **Magnetic flux:** magnetic field times area.
  Comprehensive: „Number" of magnetic field lines crossing the area delimited by a wire.
  For a homogeneous magnetic field:

  $\Phi_M = \vec{A}\cdot\vec{B} = A\cdot B\cdot\cos(\alpha)$ $\qquad \alpha = \sphericalangle(\vec{A},\vec{B})$

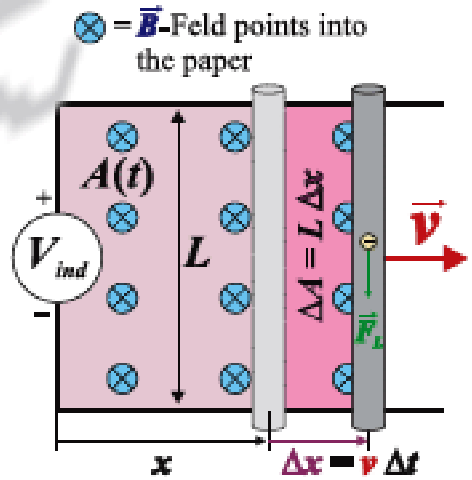
  unit: $[\Phi] = 1\,\text{Weber} = 1\,\text{Wb} = 1\,\text{V}\cdot\text{s}$.

  Particularly: (area A) $\perp$ (magnetic field B)
  $\Rightarrow\ \vec{A}\parallel\vec{B}\ \Rightarrow\ \Phi_M = A\cdot B$

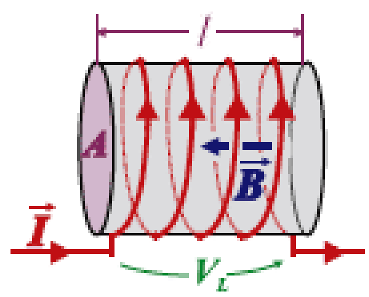


- ▶ **Lenz's law:**
  The induced voltage V_{ind} gives rise to a current whose magnetic field opposes the original change in flux.

- ▶ **Faraday's law of induction:**
  - **Induction voltage** = negative change of the magnetic flux with time (within a wire loop):
    $V_{\text{ind}} = -\frac{\Delta\Phi_M}{\Delta t} = -\frac{d\Phi_M}{dt}$
  - **Induced voltage in a coil:**
    $V_{\text{ind}} = -N\frac{\Delta\Phi_M}{\Delta t}$ $\qquad N$ = number of loops.
  - Particularly: (area A) $\perp$ (magnetic field B):
    $V_{\text{ind}} = -N\left(A\frac{\Delta B}{\Delta t} + B\frac{\Delta A}{\Delta t}\right)$



- ▶ **Inductors, inductance L, Self-inductance:**
  - Definition: $L = \frac{N\cdot\Phi_M}{I}$ $\qquad$ unit: $1\,\text{Henry} = 1\,\text{H} = 1\frac{\text{Vs}}{\text{A}}$
  - Inductance of a Solenoid of length l and cross-section area A
    $L = \mu_0\,\mu_r\cdot\frac{N^2 A}{l}$
  - Induced voltage V_L due to change in current with time:
    $V_L = -L\cdot\frac{dI}{dt} = -L\cdot\frac{\Delta I}{\Delta t}$



- ▶ **Energy stored in a coil:** $E_M = \frac{1}{2}L\,I^2$

- ▶ **Energy density of a magnetic field:** (energy density = energy per volume)
  $w_M = \frac{1}{2\mu_0\mu_r}B^2$ $\qquad$ unit: $[w_M] = \frac{\text{J}}{\text{m}^3}$

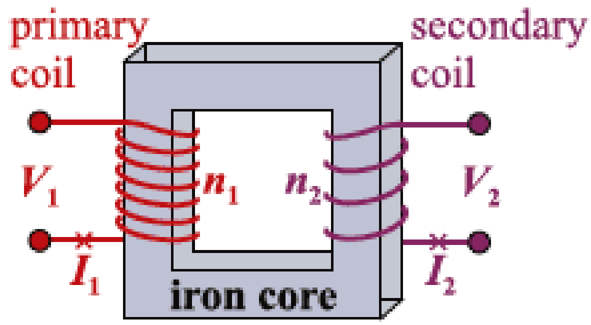
▶ Transformer (voltage converter):

Two magnetically coupled coils: $\frac{V_1}{V_2} = \frac{n_1}{n_2}$

Energy-conservation of an ideal (lossless) transformer:

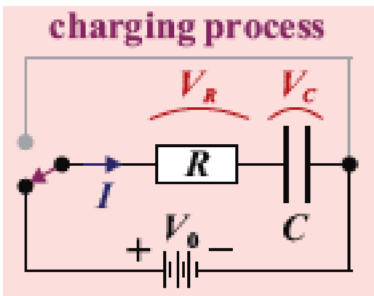
$V_1 \cdot I_1 = V_2 \cdot I_2$

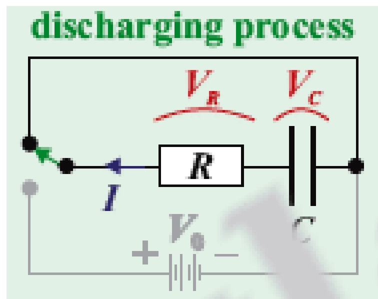
⇒ Coils see p. 23.

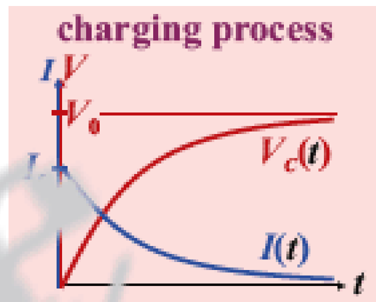


7.3 DC-Circuits

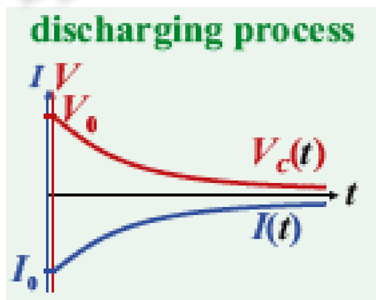
▶ RC-Circuit: charging and discharging capacitor



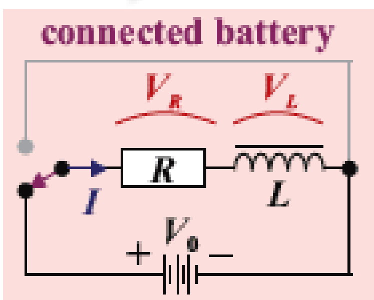


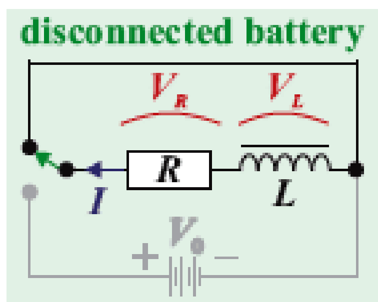


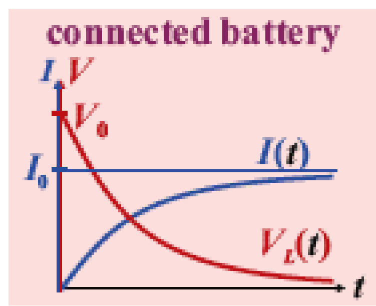
	charging process	discharging process
diff. eq.:	$R\,\frac{dQ}{dt} + \frac{Q}{C} = V_0$	$R\,\frac{dQ}{dt} + \frac{Q}{C} = 0$
charge:	$Q(t) = C\,V_0\left(1 - e^{-\frac{1}{RC}t}\right)$	$Q(t) = Q_0\,e^{-\frac{1}{RC}t}$
current:	$I(t) = \frac{V_0}{R}\,e^{-\frac{1}{RC}t}$	$I(t) = -\frac{Q_0}{RC}\,e^{-\frac{1}{RC}t}$
voltage:	$V_C(t) = V_0\left(1 - e^{-\frac{1}{RC}t}\right)$	$V_C(t) = \frac{Q_0}{C}\,e^{-\frac{1}{RC}t}$



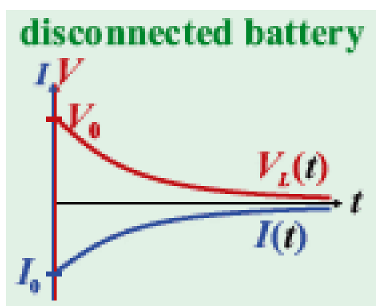
▶ RL-Circuit



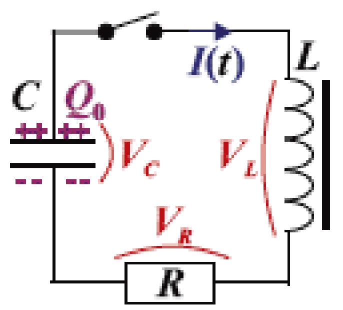


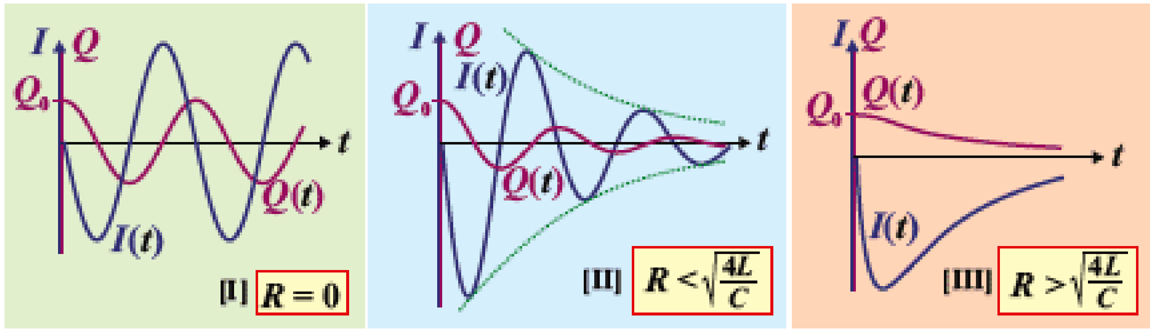


	connected battery	disconnected battery
diff. eq.:	$L\,\frac{dI}{dt} + I\,R = V_0$	$L\,\frac{dI}{dt} + I\,R = 0$
current:	$I(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$	$I(t) = -\frac{V_0}{R}\,e^{-\frac{R}{L}t}$
voltage:	$V_L(t) = V_0\,e^{-\frac{R}{L}t}$	$V_L(t) = V_0\,e^{-\frac{R}{L}t}$



▶ **RLC-Circuit: damped harmonic oscillator**





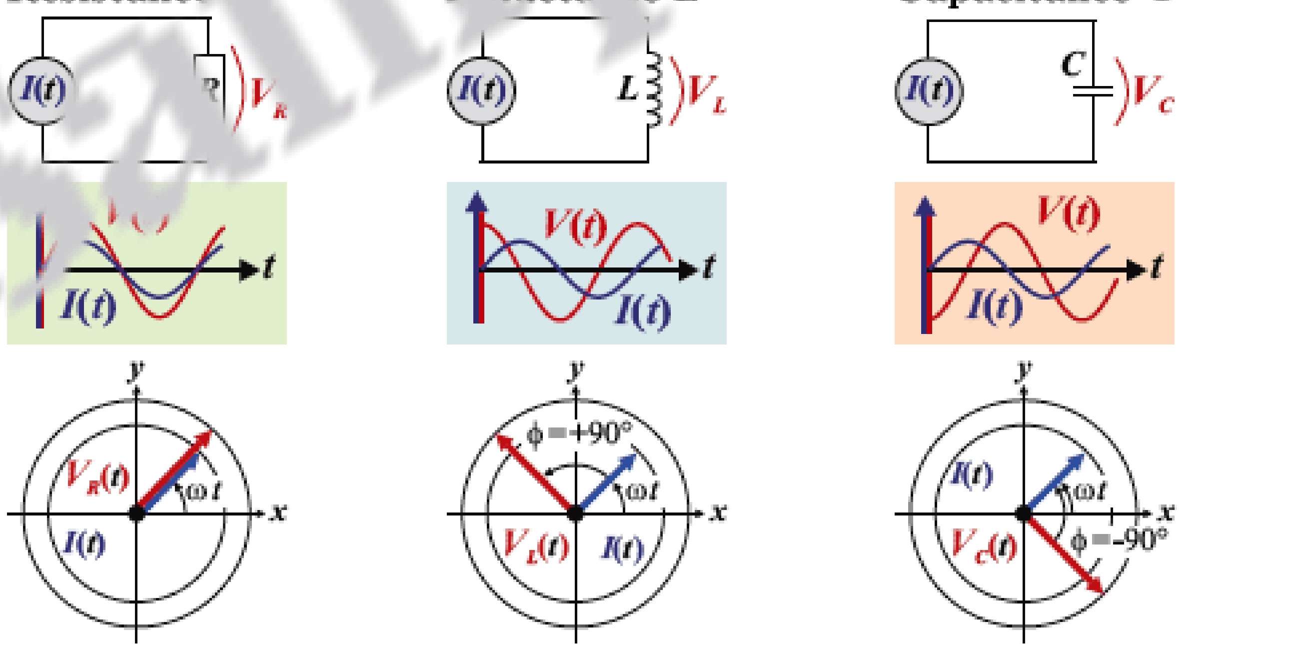
Differential equation: $L\,\frac{d^2Q}{dt^2} + R\,\frac{dQ}{dt} + \frac{Q}{C} = 0$ **Current:** $I(t) = \frac{dQ(t)}{dt}$

[I]	$R = 0$	$\Rightarrow$	$I(t) = -\,Q_0\,\omega\,\sin(\omega\,t)$	$\omega = \sqrt{\frac{1}{LC}}$
[II]	$R^2 < \frac{4L}{C}$	$\Rightarrow$	$I(t) = -\,\frac{Q_0}{\omega}\left(\omega^2 + \frac{R^2}{4L^2}\right)\,e^{-\frac{R}{2L}\,t}\sin(\omega\,t)$	$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
[III]	$R^2 > \frac{4L}{C}$	$\Rightarrow$	$I(t) = \frac{Q_0}{\omega}\left(\omega^2 - \frac{R^2}{4L^2}\right)\,e^{-\frac{R}{2L}\,t}\sinh(\omega\,t)$	$\omega = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$

7.4 AC-Circuits

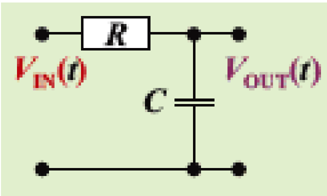
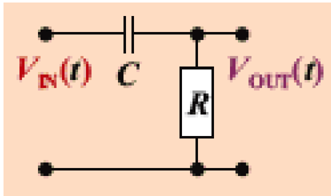
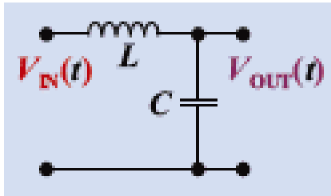
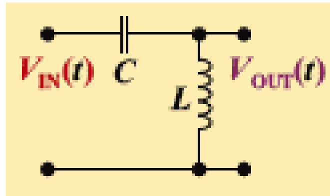
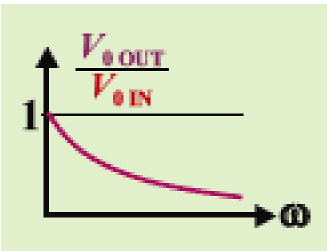
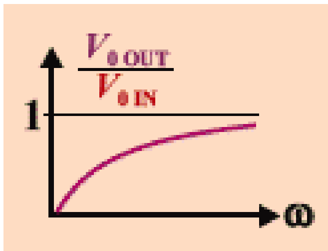
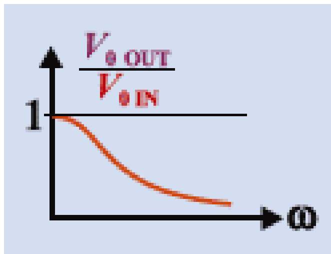
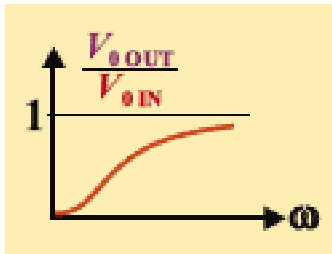
▶ **Single devices R, L, C on AC-current source** $I(t) = I_0\,\sin(\omega\,t)$

Resistance R **Inductance L** **Capacitance C**



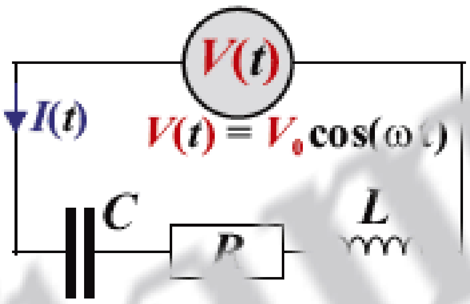
	Resistance R	Inductance L	Capacitance C
Voltage:	$V(t) = V_0\,\sin(\omega\,t)$	$V(t) = V_0\,\sin(\omega\,t + \frac{\pi}{2})$	$V(t) = V_0\,\sin(\omega\,t - \frac{\pi}{2})$
Reactance:	$(X_R = \frac{V_0}{I_0} = R)$	$X_L = \frac{V_0}{I_0} = \omega\,L$	$X_C = \frac{V_0}{I_0} = \frac{1}{\omega\,C}$
Avg. power:	$\overline{P} = R\,I_{\rm rms}^2 = \frac{1}{2}\,R\,I_0^2$	$\overline{P} = 0$	$\overline{P} = 0$

▶ RC and LC filters $V_{\text{IN}}(t) = V_0 \sin(\omega t)$

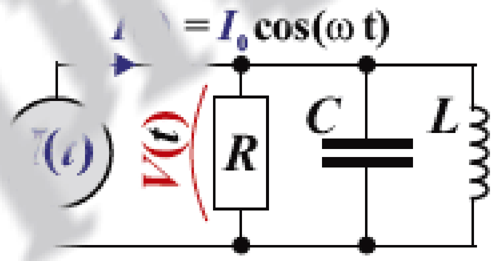
RC lowpass	RC highpass	LC lowpass	LC highpass
			
$\frac{V_{0\,\text{OUT}}}{V_{0\,\text{IN}}} = \frac{1}{\omega RC+1}$	$\frac{V_{0\,\text{OUT}}}{V_{0\,\text{IN}}} = \frac{\omega RC}{\omega RC+1}$	$\frac{V_{0\,\text{OUT}}}{V_{0\,\text{IN}}} = \frac{1}{\omega^2 LC+1}$	$\frac{V_{0\,\text{OUT}}}{V_{0\,\text{IN}}} = \frac{\omega^2 LC}{\omega^2 LC+1}$
			

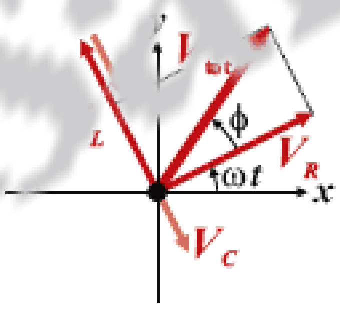
▶ RLC-oscillator

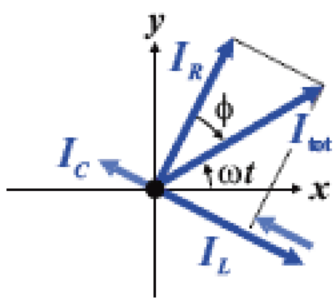
RLC series circuit



RLC parallel circuit







	RLC series	RLC parallel
Impedance:	$Z(\omega) = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$	$Z(\omega) = \left(\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2\right)^{-\frac{1}{2}}$
Voltage:	(see schematic)	$V(t) = V_0(\omega) \cdot \cos(\omega t - \phi(\omega))$
Current:	$I(t) = I_0(\omega) \cdot \cos(\omega t - \phi(\omega))$	(see schematic)
Amplitude:	$I_0(\omega) = \frac{V_0}{Z(\omega)}$	$V_0(\omega) = Z(\omega) \cdot I_0$
Phase:	$\phi(\omega) = \arctan\left(\frac{1}{R} \cdot \left(\omega L - \frac{1}{\omega C}\right)\right)$	$\phi(\omega) = \arctan\left(R \cdot \left(\frac{1}{\omega L} - \omega C\right)\right)$

⇒ Resonance phenomenon is observable on RLC series and parallel circuit, the equivalent to the mechanic resonance, see p. 12.

Condition for resonance: $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ resonance frequency: $f_0 = \frac{1}{2\pi\sqrt{LC}}$

8 Quantum physics

▶ **De Broglie relation:** Equivalence of particle and wave: $\boxed{p = \frac{h}{\lambda}}$

$p = m\,v$ = momentum, λ = wave length, $h = 6.63 \cdot 10^{-34}$ J s = Planck's constant.

▶ **Energy of a photon:** $\boxed{E = h \cdot f = \hbar \cdot \omega}$ $\quad \hbar = \frac{h}{2\pi} = 1.05 \cdot 10^{-34}$ J s

▶ **Heisenberg's uncertainty principle:**

- Position and momentum cannot have precise values at the same time: $\boxed{\Delta x \cdot \Delta p \geq \hbar}$
  Δx, Δp = uncertainty in position resp. momentum.
- Energy and time cannot have precise values at the same time: $\boxed{\Delta E \cdot \Delta t \geq \hbar}$
  ΔE, Δt = uncertainty in energy resp. time.

▶ **Schrödinger equation:** $\boxed{\frac{-\hbar^2}{2m} \cdot \frac{d^2\Psi(x)}{dx^2} + V(x) \cdot \Psi(x) = E \cdot \Psi(x)}$ (time independent)

▶ **Bohr's atomic model:**

- **Radius of electron orbit:**

  Hydrogen: $\boxed{R_n = \frac{\epsilon_0\,\hbar^2}{\pi\, m_e\, q_e^2} \cdot n^2 = a_B \cdot n^2}$

  $a_B = 5.29 \cdot 10^{-11}$ m = Bohr radius.

  In general: $\boxed{R_n \approx \frac{4\pi\epsilon_0^2\,\hbar^2}{m_e\, q_e^2\, Z}\, n^2}$

  Z = number of protons in nucleus.

- **Binding energy:**

  Hydrogen: $\boxed{E_n = -\frac{m_e\, q_e^4}{32\,\pi^2\,\epsilon_0^2\,\hbar^2} \cdot \frac{1}{n^2} = E_1 \cdot \frac{1}{n^2}}$

  $E_1 = -13.6$ eV = lowest level

  In general: $\boxed{E_n \approx -\frac{m_e\, q_e^4\, Z^2}{32\,\pi^2\,\epsilon_0^2\,\hbar^2} \cdot \frac{1}{n^2}}$

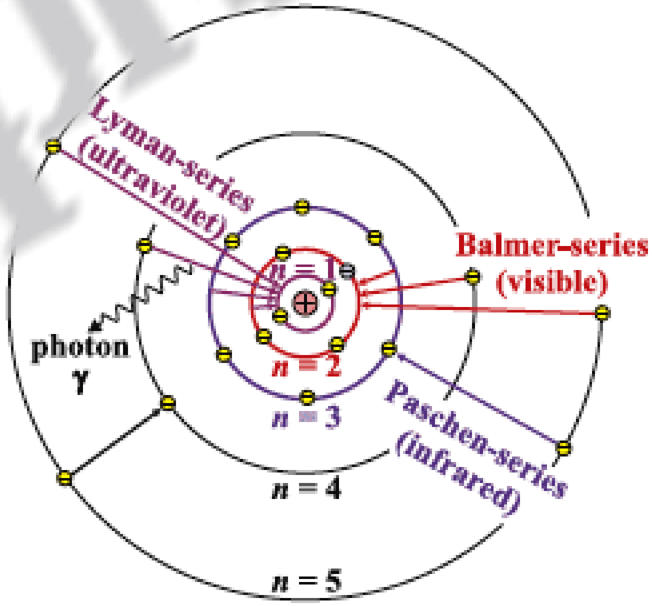
  $n = 1, 2, 3, \ldots$ principal quantum nr.

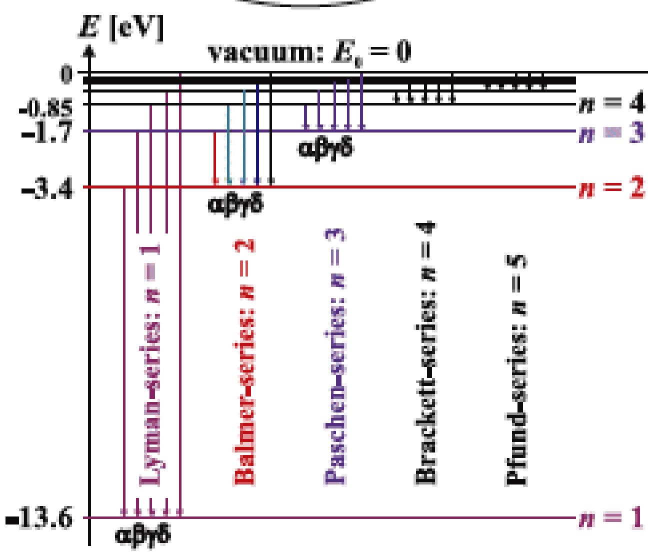
- **Frequency condition:**

  $\boxed{h\,f = |\,E_n - E_m\,|}$ $\quad E_n < E_m \Rightarrow$ absorption; $\quad E_n > E_m \Rightarrow$ emission

- **Work function (work of emission):**
  Minimum energy required to remove an electron from a solid (atom).

  $\boxed{\Phi = h\,f - \frac{1}{2} m\,v^2}$ $\quad$ (table p. 32)





⇒ Elektromagnetic spectrum on p. 33.

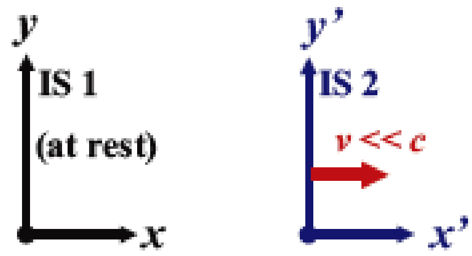
9 Special Theory of Relativity

▶ **Inertial system (IS):** Coordinate system, in which every object with mass keeps at rest or in straight motion if there is no external force acting on it. In an intertial system, Newton's second law (see p. 6) holds unlimitedly. The intertial system assumes independence of space and time.

▶ Galilei-transformation:

Transition from one inertial system to another:

$x' = x - v\,t$ (motion in x-direction)
$y' = y$
$z' = z$
$t' = t$



Any physical law is called **Galilei-invariant**, if its form is independent of the choice of the inertial system.

▶ Einstein's postulates:

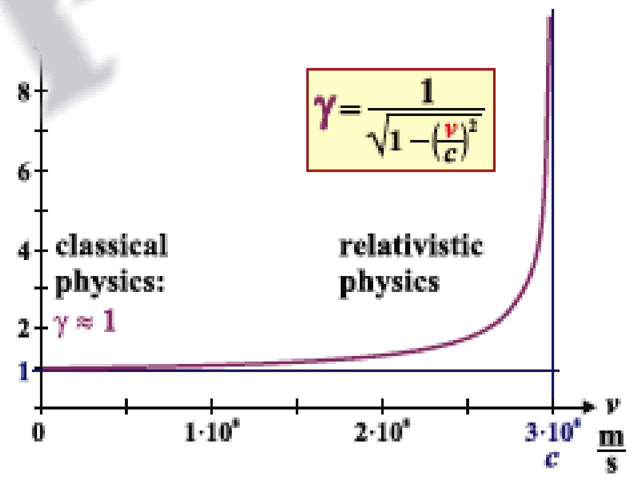
- Fundamental physical laws have the same mathematical form in all inertial systems.
- In all inertial systems the velocity of light $c = 3 \cdot 10^8\ \frac{m}{s}$ is the same and is constant, independent of direction and motion of the inertial system.

▶ Lorentz-transformation:

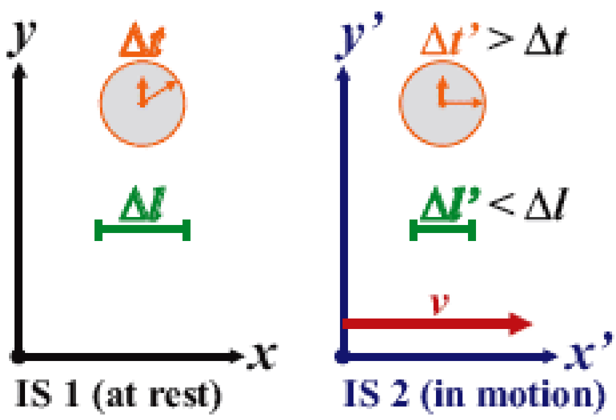
Relativistic transition from one inertial system to another: Velocity v in x-direction.

$x' = \gamma(x - v\,t)$
$y' = y$
$z' = z$
$t' = \gamma\left(t - \frac{v\,x}{c^2}\right)$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



- The 'space-time' distance is Lorentz-invariant: $x^2 + y^2 + z^2 - c^2t^2 = x_0^2 + y_0^2 + z_0^2 - c^2t_0^2$
  Consequences:
- **Length contraction:**
  "distances in motion become shorter": $\Delta l' = \frac{1}{\gamma}\,\Delta l$
- **Time dilation:**
  "clocks in motion run slower": $\Delta t' = \gamma\,\Delta t$



▶ Mass, energy and momentum:

- **Mass-energy relation:** $E = m\,c^2$
- **Relativistic momentum:** $p = \gamma\,m\,v$
- **Relativistic energy:** $E = \gamma\,m\,c^2$

10 Tables

Mechanic properties

Solids

ρ = density

E = Young's modulus

σ = stress

μ = Poisson's number

c = velocity of sound

Material	ρ in $\frac{kg}{m^3}$	E in $\frac{N}{m^2}$	σ in $\frac{N}{m^2}$	μ -	c in $\frac{m}{s}$
Aluminum (Al)	2700	$7.1 \cdot 10^{10}$	$7.1 \cdot 10^{7}$	0.34	5240
Brass	8470	$1.1 \cdot 10^{11}$	$2.9 \cdot 10^{8}$	0.35	3420
Copper (Cu)	8920	$1.2 \cdot 10^{11}$	$2.2 \cdot 10^{8}$	0.35	3900
Gold (Au)	19290	$8.2 \cdot 10^{10}$	-	0.42	3240
Iron (Fe)	7860	$\approx 2.19 \cdot 10^{11}$	$2.0 \cdot 10^{8}$	0.28	5170
Lead (Pb)	11340	$\approx 1.4 \cdot 10^{10}$	-	0.44	1250
Nickel (Ni)	8900	$2.14 \cdot 10^{10}$	$4.4 \cdot 10^{8}$	0.31	-
Platinum (Pt)	21450	$1.7 \cdot 10^{11}$	$1.4 \cdot 10^{8}$	0.39	-
Quartz (SiO_2)	2200	$9.4 \cdot 10^{10}$	-	0.17	≈ 5000
Silver (Ag)	10500	$8 \cdot 10^{10}$	$1.6 \cdot 10^{8}$	0.37	-
Tin (Sn)	7290	$4.0 \cdot 10^{9}$	-	0.33	-
Tungsten (W)	19300	4.1 · 10[illegible]	-	0.29	5460
Zinc (Zn)	7140	-	-	0.25	-
Water (ice, H_2O)	917	[illegible]	-	0.33	3250

Fluids

ρ = density

B = bulk modulus

η = viscosity (at [illegible]°C)

c = velocity of sound

Material	ρ in $\frac{kg}{m^3}$	B in $\frac{N}{m^2}$	η in $\frac{N\,s}{m^2}$	c in $\frac{m}{s}$
Acetone ($OC(CH_3)_2$)	791	-	$3.06 \cdot 10^{-4}$	1190
Benzene (C_6H_6)	879	$11.25 \cdot 10^{10}$	-	1326
Ethanol (C_2H_5OH)	789	$9.1 \cdot 10^{8}$	$1.2 \cdot 10^{-3}$	1170
Glycerol ($C_3H_5(OH)_3$)	1261	$26.6 \cdot 10^{10}$	1.48	1923
Oil	≈ 900	$16 \cdot 10^{8}$		
Petrol	850	-	-	-
Mercury (Hg)	13546	$253 \cdot 10^{10}$	$1.55 \cdot 10^{-3}$	1430
Water (H_2O)	998	$2.2 \cdot 10^{11}$	$1 \cdot 10^{-3}$	1483

Gases

ρ = density (at normal conditions)

η = viscosity (at 0°C & p = 1bar)

c = velocity of sound

Material	ρ in $\frac{kg}{m^3}$	η in $\frac{N\,s}{m^2}$	c in $\frac{m}{s}$
Air	1.293	$1.86 \cdot 10^{-7}$	344
Argon (Ar)	1.784	$2.29 \cdot 10^{-7}$	-
Butane ($CH(CH_3)_3$)	2.732	$7.5 \cdot 10^{-6}$	-
Carbon dioxide (CO_2)	1.977	$1.5 \cdot 10^{-7}$	268
Freon	5.51	-	-
Helium (He)	0.1785	-	1005
Hydrogen (H)	0.0899	-	1310
Methane (CH_4)	0.717	$1.12 \cdot 10^{-7}$	445
Neon (Ne)	0.9	-	-
Nitrogen (N_2)	1.25	-	337
Oxygen (O_2)	1.429	-	326

Friction coefficients:

Material	μ_S (static)	μ_K (kinetic)	μ_R (rolling)
wood on wood	0.6	0.4	-
steel on steel	0.15	0.1	≈ 0.002
rubber on tar (bitumen)	1.0	0.6	≈ 0.02
steel on ice	0.027	0.014	-

Thermal data

Solids

α = coefficient of linear thermal expansion
c = specific heat capacity
T_{MP} = melting point (at normal pressure)
L_f = specific latent heat of fusion
k = thermal conductivity

Material	α in $\frac{1}{\mathrm{K}}$	c in $\frac{\mathrm{J}}{\mathrm{kg\,K}}$	T_{MP} in °C	L_f in $\frac{\mathrm{J}}{\mathrm{kg}}$	k in $\frac{\mathrm{W}}{\mathrm{m\,K}}$
Aluminum (Al)	$23.8 \cdot 10^{-6}$	896	660.1	$3.97 \cdot 10^5$	239
Brass	$18 \cdot 10^{-6}$	380	905	$1.6 \cdot 10^5$	79
Copper (Cu)	$16.8 \cdot 10^{-6}$	383	1083	$2.05 \cdot 10^5$	[illegible]
Gold (Au)	$14.3 \cdot 10^{-6}$	129	1063	[illegible]	[illegible]
Iron (Fe)	$12.0 \cdot 10^{-6}$	450	1535	[illegible]	[illegible]
Lead (Pb)	$31.3 \cdot 10^{-6}$	129	[illegible]	[illegible]	[illegible]
Nickel (Ni)	$12.8 \cdot 10^{-6}$	448	[illegible]	[illegible]	81
Platinum (Pt)	$9.0 \cdot 10^{-6}$	[illegible]	[illegible]	[illegible]	70.1
Quartz (SiO_2)	$4.5 \cdot 10^{-7}$	710	[illegible]		1.36
Silver (Ag)	[illegible]	[illegible]	[illegible]	$1.045 \cdot 10^5$	428
Tungsten (W)	[illegible]	[illegible]	[illegible]	$1.92 \cdot 10^5$	177
Tin (Sn)	[illegible]	[illegible]	231.9	$0.596 \cdot 10^5$	64
Water (ice H_2O)	[illegible]	2100	0	$3.338 \cdot 10^5$	2.2
Zinc (Zn)	[illegible]	385	419.5	$1.11 \cdot 10^5$	112

Fluids

γ = coefficient of volume expansion
c = specific heat capacity
T_{MP} = melting point (at normal pressure)
T_{BP} = boiling point (at normal pressure)
L_f = specific latent heat of fusion
L_v = specific latent heat of vaporisation
k = thermal conductivity

Material	γ in $\frac{1}{\mathrm{K}}$	c in $\frac{\mathrm{J}}{\mathrm{kg\,K}}$	T_{MP} in °C	T_{BP} in °C	L_f in $\frac{\mathrm{J}}{\mathrm{kg}}$	L_v in $\frac{\mathrm{J}}{\mathrm{kg}}$	k in $\frac{\mathrm{W}}{\mathrm{m\,K}}$
Acetone ($OC(CH_3)_2$)	$1.49 \cdot 10^{-3}$	2160	-94.86	56.25	$9.8 \cdot 10^4$	$5.25 \cdot 10^5$	0.162
Benzene (C_6H_6)	$1.23 \cdot 10^{-3}$	1725	5.53	80.1	$1.28 \cdot 10^5$	$3.94 \cdot 10^5$	0.148
Ethanol (C_2H_5OH)	$1.1 \cdot 10^{-3}$	2430	-114.5	78.33	$1.08 \cdot 10^5$	$8.4 \cdot 10^5$	0.165
Glycerol ($C_3H_5(OH)_3$)	$5.0 \cdot 10^{-4}$	2390	18.4	290.5	$2.01 \cdot 10^5$	$8.54 \cdot 10^5$	0.285
Mercury (Hg)	$1.84 \cdot 10^{-4}$	139	-38.87	356.58	$1.18 \cdot 10^4$	$2.85 \cdot 10^5$	8.2
Water (H_2O)	$2.07 \cdot 10^{-4}$	4182	0	100	$3.338 \cdot 10^5$	$2.256 \cdot 10^6$	0.598

Gases

c_p = specific heat capacity at p = constant
C_p = molar heat capacity at p = constant
$\kappa = \frac{C_p}{C_V}$
T_{MP} = melting point (at normal pressure)

T_{BP} = boiling point (at normal pressure)
L_f = specific latent heat of fusion
L_v = specific latent heat of vaporisation
a, b = Van-der-Waals constants

Material	c_p in $\frac{\mathrm{J}}{\mathrm{kg\,K}}$	C_p in $\frac{\mathrm{J}}{\mathrm{mol\,K}}$	κ -	T_{MP} in °C	T_{BP} in °C	a in $\frac{\mathrm{N\,m^4}}{\mathrm{mol^2}}$	b in $\frac{\mathrm{m^3}}{\mathrm{mol}}$
Argon (Ar)	523	20.9	1.305	−77.7	−33.4	0.425	$3.73 \cdot 10^{-5}$
Freon	502	60.7	1.14	−158.2	−29.8	0.837	$7.75 \cdot 10^{-5}$
Helium (He)	5230	20.9	1.63	-	−268.94	0.0034	$2.36 \cdot 10^{-5}$
Carbon dioxide (CO_2)	837	36.8	1.293	-	−78.45	0.366	$4.28 \cdot 10^{-5}$
Air	1005	29.1	1.402	-	−191.4	0.135	$3.65 \cdot 10^{-5}$
Methane (CH_4)	2219	35.6	1.402	-	−191.4	0.229	$4.28 \cdot 10^{-5}$
Neon (Ne)	1031	20.8	1.64	-248.61	−246.06	0.0217	$1.74 \cdot 10^{-5}$
Oxygen (O_2)	917	29.3	1.398	−218.79	−182.97	0.138	$3.17 \cdot 10^{-5}$
Nitrogen (N_2)	1038	29.1	1.401	−210.0	−195.82	0.137	$3.87 \cdot 10^{-5}$
Water vapor (H_2O)	1863	33.6	1.33	0	1[illegible]	0.553	$3.04 \cdot 10^{-5}$
Hydrogen (H)	14320	28.9	1.41	−[illegible]9.2	[illegible]252.77	0.0248	$2.66 \cdot 10^{-5}$

Electric, magnetic and optical data

Electric conductors (metals)

ρ = specific resistance (at [illegible]0°C)
α = temperature coefficient

μ_r = magnetic permeability
Φ = work function

Material	ρ in $\Omega \cdot \mathrm{m}$	α in $\frac{1}{\mathrm{K}}$	μ_r -	Φ in eV
Aluminum (Al)	$2.82 \cdot 10^{-8}$	$+3.9 \cdot 10^{-3}$	$1 + 2.1 \cdot 10^{-5}$ (paramagnetic)	4.2
Brass	$7 \cdot 10^{-8}$	$+2 \cdot 10^{-3}$		-
Caesium (Cs)				1.87
Germanium (Ge)	0.14	-	-	-
Copper (Cu)	$1.7 \cdot 10^{-8}$	$+3.9 \cdot 10^{-3}$	$1 - 6.4 \cdot 10^{-6}$ (diamagnetic)	4.84
Gold (Au)	$2.2 \cdot 10^{-8}$	$+4 \cdot 10^{-3}$	$1 - 3.4 \cdot 10^{-5}$ (diamagnetic)	4.83
Iron (Fe)	$1 \cdot 10^{-7}$	$+5 \cdot 10^{-3}$	≈ 5800 (ferromagnetic)	-
Lead (Pb)	$2.2 \cdot 10^{-7}$	$+3.9 \cdot 10^{-3}$	diamgnetic	-
Potassium (K)	-	-	-	2.15
Nickel (Ni)	$7.8 \cdot 10^{-8}$	$+6 \cdot 10^{-3}$	≈ 1120 (ferromagnetic)	5.09
Platinum (Pt)	$1 \cdot 10^{-7}$	$+3 \cdot 10^{-3}$	$1 + 2.8 \cdot 10^{-4}$ (paramagnetic)	5.3
Silver (Ag)	$1.59 \cdot 10^{-8}$	$+3.8 \cdot 10^{-3}$	-	4.43
Tungsten (W)	$5.3 \cdot 10^{-8}$	$+4.8 \cdot 10^{-3}$	-	4.57
Zinc (Zn)	$5.8 \cdot 10^{-8}$	$+3.7 \cdot 10^{-3}$	-	4.34

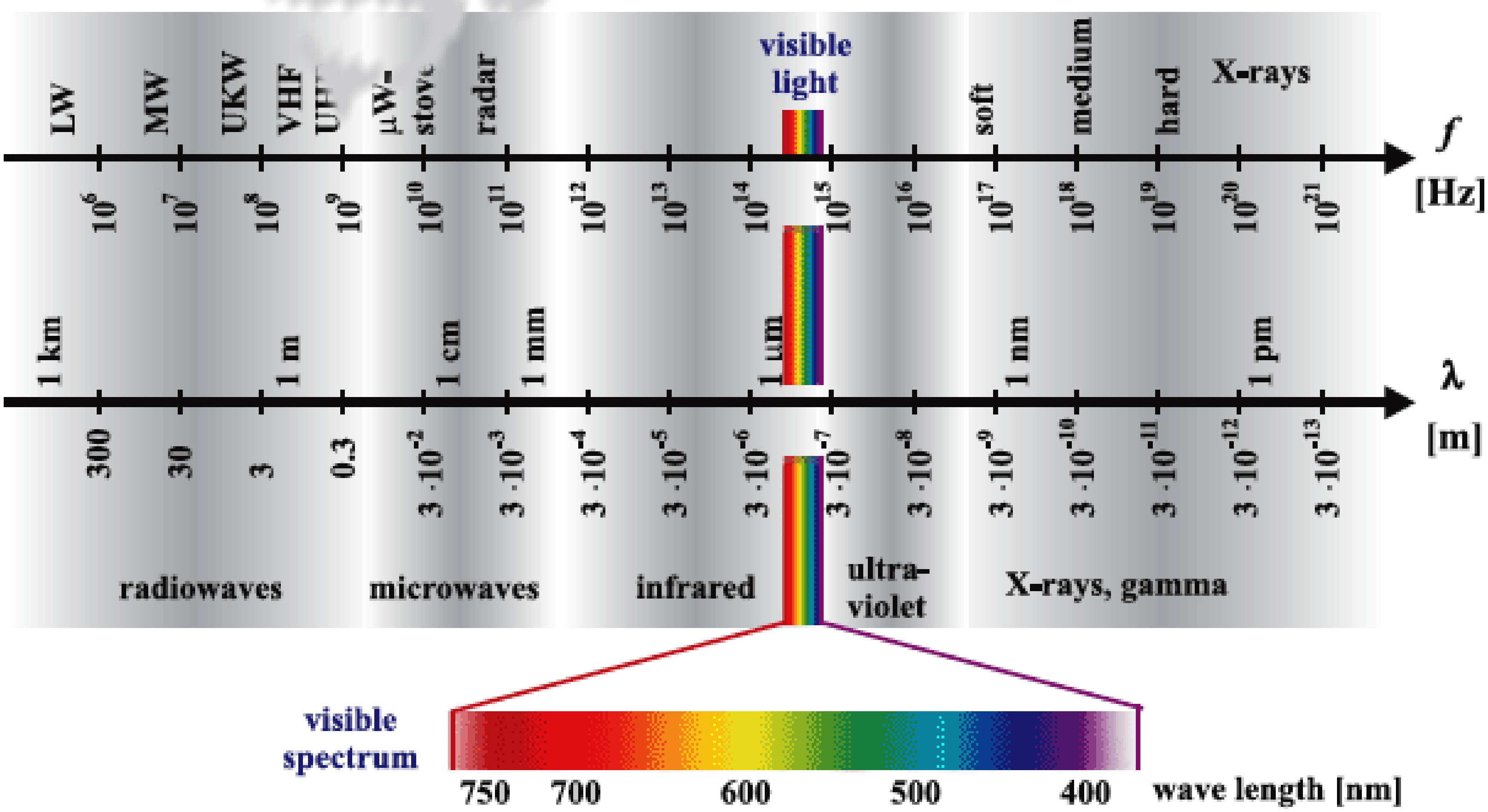
Elektric insulators, transparent materials

ρ = specific resistance (at 20°C)
ϵ_r = permittivity (dielectric constant)

n = index of refraction (towards vacuum)

Material	ρ in $\Omega \cdot m$	ϵ_r -	n -
Air	-	1.0006	1.000272
Benzene (C_6H_6)	-	2.3	1.49
Diamond (C)	-		2.42
Ethanol (C_2H_5OH)	-	-	1.36
Glycerol ($C_3H_5(OH)_3$)	-		1.47
Ice (H_2O)	-		1.31
Mica	$5 \cdot 10^{14}$	7	-
Paraffin	$3 \cdot 10^{16}$	2.1	-
Plexiglas	$1 \cdot 10^{13}$	3.4	1.5
Quartz (SiO_2	$3 \cdot 10^{14}$	4	1.46
Salt (NaCl)	-		1.54
Silicium (Si, pure)	$1.7 \cdot 10^4$		-
Teflon	$1 \cdot 10^{13}$	2	-
Water (H_2O)	-	80	1[illegible]3

Electromagnetic spectrum



Astronomical data

m = mass of the celestial body
r = radius of the celestial body
T_C = time of circulation
a bzw. R = semimajor axes resp. radius of circulation
T_{Rot} = Time for one revolution
g = acceleration of fall
v_F = escape velocity

Celestial body	m in kg	r in m	T_C in days	a resp. R in m	T_{Rot}	g in $\frac{\text{m}}{\text{s}^2}$	v_F in $\frac{\text{km}}{\text{s}}$
Mercury	$3.31 \cdot 10^{23}$	$2.425 \cdot 10^{6}$	87.969	$5.79 \cdot 10^{10}$	58.65 d	3.63	4.2
Venus	$4.87 \cdot 10^{24}$	$6.070 \cdot 10^{6}$	224.701	$1.082 \cdot 10^{11}$	243 d	8.60	10.3
Earth	$5.98 \cdot 10^{24}$	$6.378 \cdot 10^{6}$	365.256	$1.496 \cdot 10^{11}$	23.93 h	9.81	11.2
Mars	$6.42 \cdot 10^{23}$	$3.395 \cdot 10^{6}$	686.98	$2.279 \cdot 10^{11}$	24.63 h	3.74	5.0
Jupiter	$1.90 \cdot 10^{27}$	$7.13 \cdot 10^{7}$	4332.57	$7.783 \cdot 10^{11}$	9.48 h	25.9	61
Saturn	$5.69 \cdot 10^{26}$	$6.01 \cdot 10^{7}$	10759.22	1.4[illegible] $\cdot 10^{12}$	10[illegible] h	11.3	37
Uranus	$8.69 \cdot 10^{25}$	$2.56 \cdot 10^{7}$	30685.40	2.875 [illegible]0^{12}	17.2 h	9.0	22
Neptun	$1.03 \cdot 10^{26}$	$2.43 \cdot 10^{7}$	6018[illegible]	4.50[illegible]	15.8 h	11.5	24
Moon	$7.35 \cdot 10^{22}$	$1.74 \cdot 10^{6}$	[illegible]32	[illegible].84[illegible] $\cdot 10^{8}$		1.622	2.38
Sun	$1.99 \cdot 10^{30}$	6.9[illegible] 10^{8}	-	-	-	273.98	617.7

Perso[illegible]es

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